

On Sraffa's "Corrected" Organic Composition of Capital¹

In February 1944 Sraffa made a major breakthrough that would lead him to the eventual construction of what would emerge as his Standard system, the Standard ratio, and the Standard commodity. This would take the form of a "correction" (Sraffa's word) of the organic composition of capital (OCK) concept found in Marx. The arrival at the "corrected" version of the OCK in early 1944 was preceded by several years of intense intellectual activity, beginning in 1940 with his re-reading of *Capital*, Volume I while interned on the Isle of Man for a three month period (months???) and re-commencing in full force in August 1942 with the evolution of his "equations" that he had developed from the late 1920s and early 1930s in the first period of intense scientific activity.² During the period beginning in 1940, interrupted in 1941 by lecture duties, and recommencing with vigor beginning August 1942, Sraffa developed his equation systems for a variety of situations and scenarios and conceptualized in various degrees of evolution his "Hypothesis" regarding a fundamental constancy in his economic system. He also thoroughly engaged the writing of various authors such as Laidius von Bortkiewicz, Eugene Böhm-Bawerk, Knut Wicksell, Erik Lindhal, Rangar Nurkse, Jacob Marschak, Gustav Cassel, Irving Fisher, Friedrich von Hayek, von Weiser, Arthur Cecil Pigou, J.B. Clark, and Joan Robinson, to name some of those most important. His notes on Bortkiewicz are especially rich and have been thoroughly dealt with in Gerkhe and Kurz (2006). During this period he worked out his equation systems in terms of what he calls "discontinuous" and "continuous" economic systems, the former named "Agricultural schemes" and the latter "Industrial schemes". In terms of modern parlance, the former equation system is of the point-input point-output variety, and the latter the flow-input point-output variety.³ And it was with respect to the continuous flow of inputs in the Industrial schemes that Sraffa spent

¹ Scott Carter, The University of Tulsa. Paper written for 'Sraffa's *Production of Commodities by Means of Commodities*' 1960-2010: *Critique and Reconstruction of Economic Theory*, Rome, Italy, December 2-4, 2010. Thanks to Professor Garegnani for quotation from the Sraffa Papers and Riccardo Bellofiore for everything else. Please note that the sole responsibility for interpretations is the author's. It should be noted that any curly brackets {} inserted into Sraffa's archival material are ours, whereas rounded parentheses () and square brackets [] are those used by Sraffa. The author would also like to thank Jonathan Smith, Archivist at Trinity College, University of Cambridge. Note that this is a preliminary unfinished draft. Please do not quote.

² It is well known and well documented (Smith 1998, Naldi 2005, Gilibert 2003, Bellofiore 2008, Garegnani 2005, Kurz 2006, Gerkhe and Kurz 2006, etc...) that Sraffa engaged in three periods of intense scientific activity as regards the constructive development of the content of *Production of Commodities by Means of Commodities* (hereafter referred alternatively as his "book"): the first period from 1927 to 1931-2; the second period from 1940; 1942-1945-6, and the third and last period from 1955-1959. In the following essay we concentrate on the second period in the 1940s. We mark 1940 as the beginning of this second period due to his re-reading of *Capital* and notes he penned in November 1940 entitled "Use of the notion of Surplus-Value" (D3/12/46/57-63), both of which we concur with Bellofiore (2008) had a major impact on the subsequent intense activity he would again commence beginning in August 1942. The interim between December 1940 and August 1942 saw Sraffa engaged in his war-time Lectures on Industry (D2/8); on the latter see Marcuzzo (2005).

³ Howard (1980, p. 3) identifies four different types of models, moving from the most general to the most restrictive: (1) the flow-input flow-output model; (2) the flow-input point-output model; (3) the flow-input flow-output model; and (4) the point-input point-output model. Like Sraffa's models in the period of the 1940s, Howard notes that "the earliest Austrian capital theorists, Jevons, Böhm-Bawerk, and Wicksell, concentrated their analysis on the latter two types" (p. 3). For a thorough comparison of these alternative conceptualizations of such processes especially as regards the relationship to Classical political economy, see especially Lager (2000) and the entire Special Issue of *Economic Systems Research* where it can be found.

tremendous intellectual energy on the Austrian reduction methodology, a methodology that he found both very powerful but also very “dangerous”⁴ and subject to significant limitations.

{ANOTHER INTRODUCTORY PARAGRAPH}

The remainder of this essay is as follows. In the first section Sraffa’s Hypothesis will be briefly discussed. Following a thesis first advanced by Gilibert (2003) and developed further by Bellofiore (2008, 2010), it will be shown that Sraffa conceived of the constancy his Hypothesis (or “Hypo”) assumed at least as early as August 1942 in notes entitled CROSSCAP, and further that the specter of Marx looms heavy here. The second section traces the development of Sraffa’s various equation systems from Agricultural schemes to Industrial Schemes and his treatment of the Reduction. It will be shown in this section that critical here was the utilization of the Hypothesis in Sraffa’s consequent conceptualization of what would eventual flower into the Standard commodity via the evolution through the concepts of the “Average commodity” to the “All commodity” and eventually to a composite Standard commodity. The third section shows that this was all possible due to a fundamental “correction” that Sraffa made to the organic composition concept found in Marx, the correction being the conceptualization of the “new” composition such that means of production are conceived in terms of the homogenous commodity inputs necessary economy-wide as opposed to the “old” conception that involved the heterogeneous means of production input requirements per industry; whereas the latter (“incorrect”) conceptualization requires a principle of valuation given the heterogeneity of the means of production requirements, the former does not since both the numerator and denominator consist of physical units of the same commodity – in a word, Sraffa’s “corrected” OCK allows him to *not* be “stopped by the word ‘price’”⁵. The final section concludes.

The present essay builds on several groundbreaking interpretations of Sraffa’s inquiries in the period of the 1940s based on archival research. The most influential of these interpretations are Gilibert (2003), de Vivo (2003), Bellofiore (2008, 2010) – although it must be mentioned that the later dates of these publications does not reflect the fact that the thesis Bellofiore advances had been in the works since the late 1990s; Kurz (2006); Kurz and Gerkhe (2006), and Kurz and Salvadori (2008, 2010). This essay is intended to contribute to this important literature by recounting the development of the concept of the organic composition of

⁴ In a series of notes entitled “Closed Vertical Combines” (D3/12/22) written in October 1942 immediately after Sraffa developed his Agricultural Schemes (D3/12/19) of August 1942 and his Industrial Schemes (D3/12/21) of September 1942, and which we argue below represents a sort of melding of the two schemes, Sraffa notices that “closing the combine” leads to a large degree to assumptions made by Böhm-Bawerk, to which he cautions that:

“N.B. The dangers of this trap, which are very great, can be seen if we consider that it leads to assumptions equivalent to those of B-B {Böhm-Bawerk} (reduction of all capital to labour); but without the patent absurdity of his schemes which supposes that ‘ultimately’ all goods are produced by naked hands” (D3/12/22/1 : 2).

We will return to the importance for the integrated combine model for Sraffa below.

⁵ The reference here is to David Ricardo’s letter to James Mill in December 1815. We read from Sraffa’s Introduction to *Works I*:

“It is remarkable that in [the] letters of October and November 1815 which give the main headings of the proposed work (Rent, Profit, Wages) there is no reference to Value. This is mentioned for the first time, as a separate subject with which it occurred to Ricardo that he would have to deal, in a letter to Mill of 30 December. ‘I know I shall be soon stopped by the word price,’ he writes, ‘and then I must apply to you for advice and assistance. Before my readers can understand the proof I mean to offer, they must understand the theory of currency and of price.’ From this time onwards the problem of Value increasingly troubled him. On 7 February 1816 he writes to Malthus: ‘If I could overcome the obstacles in the way of giving a clear insight into the origin and law of relative or exchangeable value I should have gained half the battle.’” (*Works I*, pp. xiv-xv).

capital that Sraffa engaged during the period of the 1940s leading up to the full-fledged blossoming of the Standard commodity.

I. Sraffa's "Hypothesis"

One of the upshots of Sraffa's inquiries during this period was his conceptualization of the notion of a fundamental constancy in his economic system, what he referred to as his "Hypothesis" or "Hypo" (see Gilibert 2003; de Vivo 2003; Bellofiore 2008, 2010; Kurz 2006; Gerhke and Kurz 2010; Kurz and Salvadori 2008, 2010). The main thrust of his Hypothesis was the idea that the value of social capital relative to social product remains constant in the face of changes in distribution. As both Gilibert (2003, p. 36) and de Vivo (2003, p. 16) note with surprise, the idea that social capital relative to social product would be constant given changes in prices seems at first sight to be "most un-Sraffian"; Sraffa himself would in this period abandon the notion as it was originally conceived (he refers to the "disaster of the model"; see especially Gilibert 2003, pp. 38-39), although the idea of invariance in the face of distributional changes would remain and eventually flower into the Standard commodity. And this is precisely the point – Sraffa's "Hypothesis" during this period would provide for him an imperfect yet insightful way to conceive of the price-theoretic implications of changes in income distribution. Throughout the period leading up to his "correction" of the organic composition of capital in February 1944, the Hypo provided a powerful organizing and conceptualizing tool that allowed Sraffa to explore what he discerned as fundamental characteristics of the distribution problematic in capitalistic socio-economic systems.

Before moving to the Hypo itself, we must first develop the line of analysis that Carter (2009, 2010) and Bellofiore (2008) take regarding the importance of Sraffa's re-reading of *Capital*, Volume I in summer 1940. In Carter (2009, 2010), archival evidence is presented that underscores the importance the rate of exploitation had for the manner in which Sraffa conceives of the problem of price-value deviations in the face of changes in income distribution. From notes written and inserted on an index card in his copy of *Capital*, Sraffa develops the notion of that the *source* of the deviation between value and price emanates from the extraction of unpaid labor, subsequently amassed in an abstract notion he would from 1942 clear through to December 1955 call the "pool of profits". In 1940 Sraffa refers to this mass of extracted unpaid labor as "snow":

"The greater the degree of exploitation in a society as a whole, the greater is the distortion (i.e. the divergence between values and prices). As, the greater the amount of snow fallen, the greater is the distortion of the surface of a piece of broken ground (i.e. divergence between the surface of the snow and that of the ground underneath; since the snow collects in the cavities)" (Sraffa 3731).

In Carter (2009) we document the development of Sraffa's "snow" into the idea of the "pool of profits". There and in Carter (2010) it is shown that the "profits pool" notion became a fundamental driver in the development of what would later become surplus and deficit industries in Sraffa's book. The basic idea here is simply that given different conditions of production as expressed as different ratios of labor to means of production⁶, as the wage share falls the mass of unpaid labor gets distributed to the owners of capital as if it were doled out of a common fund, or "pool", according the value of the capital in the industry.

⁶ The labor to means of production or LMP ratio is the expression of the organic composition of capital concept that made it into Sraffa's book. As is well known, it is defined as the value of the living labor added in each industry divided by the value of the (heterogeneous) means of production requirements of that industry. In more-than-one commodity models by definition the LMP ratios will differ, with relatively labor-intensive industries (LI) having an LMP ratio greater than that of means of production-intensive (MPI) industries. Labor-intensive industries will be what Sraffa refers to as

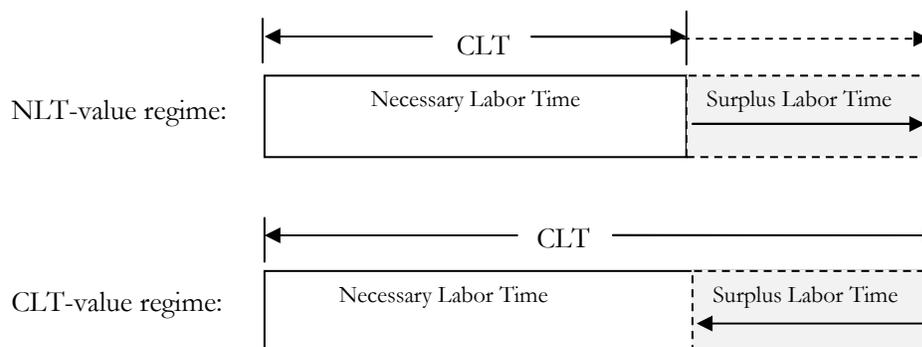
The idea of a mass of extracted unpaid labor is also considered by Sraffa in notes he penned in November of 1940 entitled “Use of the notion of Surplus Value” (D3/12/46/58- 63). As shown in Bellofiore (2008, 2010)⁷, it is in these notes that Sraffa turns the table on the idea of a decrease in the share of wages and increase in rate of profits and instead conceives of an economic system wherein the working day is actually “shortened” such that the entire living labor added is just enough to pay the necessary portion of the working day. Here we find an interesting “reverse-analogue” of the subsistence model he would develop in Chapter I of his book. This allowed for the conceptualization of the price phenomenon strictly in terms of “untransformed” *values*, as opposed to “transformed” *prices of production*:

“Now, we are comparing the actual state with a hypothetical one in which only the necessary labour is performed. In the actual state commodities are exchanged at their prices, whilst in the hypothetical state (where there would be nothing to be paid out of profits) at their values” (D3/12/46/58).

This greatly influenced the manner in which Sraffa conceived of the price form necessary to establish the constancy condition of his Hypothesis, the difference here being that instead of “subsistence” values where the working day is shortened to the lower limit, Sraffa would in his subsequent notes consider “complete” values such that the entire value added is commanded by living labor at the upper limit. We may conceive of the “lower limit” values as *necessary labor-time values* (NLT-values) and the “upper limit” values as *complete labor time values* (CLT-values). Formally the two “values at the extremes” are equivalent in the sense that living labor commands its entire product and the rate of profit is zero as no unpaid labor is extracted in either case. The difference between the two lay in the mechanism wherein unpaid labor manifests and profit thus arises. For NLT-values the working-day must be lengthened while the necessary portion remains constant in order for profit to arise; extra labor-time is “added” to the total living labor performed. For CLT-values the living labor added hence to total value-added remains constant and the necessary portion of the working day decreases.

surplus industries (SI), in that the extracted unpaid labor there exceeds that which is necessary to distribute – from the “pool” - as profits to the owners of that industry, and vice versa for capital-intensive industries, which Sraffa refers to as deficit industries (DI). As also is well known, there will be a “watershed” LMP ratio that marked the distinction between LI *qua* SI vs. MPI *qua* DI; this “watershed ratio” will be the Standard ratio which is “coincidentally” equal to the maximum rate of profit, with both ratios denoted by the single letter “R”. See Carter 2009, 2010 for a fuller account of this argument.

⁷ Bellofiore (2008) uses this idea in Sraffa to make the important distinction between the temporal quality of the living labor, and reads into this ramifications for Marx’s labor theory of value, specifically the “Sraffian” rejection of it. He asserts that “Sraffa’s object of analysis is a typically *Ricardian* one” such that he (Sraffa) “begin(s) his discourse when the process of production has *ended*, and living labour is now *dead* in the commodity” (p. 80; emphasis in text). Bellofiore defines this as “looking at the economic system *post factum* [which] cannot but make Marx’s labour theory of value *redundant*” (p. 81; emphasis in text). What we may call the *ant factum* view of living labor is characterized by Bellofiore as follows: “Surplus value exists *only* as long as labour *in motion* exceeds the labour required to reproduce the worker” and assert that “[a]s a matter of fact, Sraffa stops here” (p. 80; emphasis in text). We find this *ante factum* vs. *post factum* qualitative distinction between living labor very interesting and are of the opinion that the former is not at odds with the theoretical structure of Sraffa’s own analysis.

Figure 1: Accrual of Profit in NLT-values vs. CLT-values

In both scenarios the quantum of living labor is commanded by the workers in its entirety, unpaid labor extracted is nil, and the rate of profits is zero. This of course strikes of Marx's own distinction in Volume I of *Capital* between the extraction of absolute vs. relative surplus value.

What is important for our purposes is not which of these no-profit values (prices) are "correct"; rather what is important is the idea that no-profit values (prices) are conceived in the first place. Sraffa of course would in subsequent notes revert to the CLT-value structure by explicitly considering the economic system solved for the system of prices such that the wage share is unity and the wage rate is "complete" in the sense defined by Pasinetti (1977).⁸ Sraffa utilized the set of prices associated with the "complete wage rate" (unitary wage share) throughout both the 1940s and 1950s, and in the 1950s coined such a procedure as the "value theory of labour":

21.2.1955

In the dust raised by the controversies on the Labour Theory of Value, a valuable {above "interesting"} aspect has been overlooked, or what be called the Value theory of Labour. Wicksell would have benefited from it – it would have answered some of its puzzles. For, whatever disputes there may be about the determination of value by the quantity of labour, there can be no doubt (nobody doubts so far as I know) that the value of a commodity (its price for $r = 0$) determines (i.e. measures) the quantity of labour which directly or indirectly has entered into its production." (D3/12/44/3).

This exposition of the "value theory of labour", written in February 1955 only a few weeks before his trip to Majorca and the subsequent writing of the very important first draft of Part I of his book on single product industries he refers to as the Majorca Draft (D3/12/52), is written on notes that Sraffa had originally penned in December of 1946.⁹ Clearly Sraffa had in 1955 returned to his notes from nine years before with a mind to

⁸ "An 'ideal' system of prices...determines...relative prices and a wage rate which absorbs the entire net product per worker in the economic system. This [is] regarded as the 'maximum' wage rate...since it corresponds to a profit rate of zero. We may call it the 'ideal' wage rate...or, from a different point of view, the 'complete' wage rate..."(Pasinetti, 1977, p. 122).

⁹ Gehrke and Kurz (2006), Kurz (2006) and Kurz and Salvadori (2010) also develop the importance of the "value theory of labour" in Sraffa, although they date the coinage of this term to the mid to late 1940s whereas we are of the opinion that the particular term (VTL) was coined in 1955. Irrespective of when the term was introduced, Sraffa certainly worked with unitary wage share price systems as far back as the late 1920s when he was developing his equations, and especially operationalized it in the early 1940s.

finally put his inquiries into publishable form. These notes from 1946 relate explicitly to the present question at hand:

D3/12/44/3

“Dec 46

A Definite Result

Extension of the meaning of, and measure of, the ‘quantity of labour contained in a commodity’

The steps are:

1. Produced by a day’s labor, and nothing else (this is the starting point)
2. Produced by a day’s labour, and material which in turn is produced by labour alone (or, finite series)
3. Produced by labour and material produced by labour and material, and so on... (infinite series – sum of) (wheat)
4. Here difficulties fork:

We cannot decide whether ‘fixed capital’ or ‘joint prods.’ is ‘simpler’ because their difficulties are of a different kind. [So far, the problem was only to measure quantity of labour, and value; now we must meet that of dating that labour; and even that is insufficient for fixed capital. We must date the product too.

We discover that Sraffa locates the idea of the “value theory of labour”, understood in this context as the set of prices ascertained when the wage share is unity (Pasinetti’s “complete wage rate” system), within the context of his approach to the reduction to quantity of dated labour. And it is here that we can understand how the “value theory of labour” in fact “answered some of [Wicksell’s] puzzles”.¹⁰

Wicksell’s “puzzles”

We begin with an interesting document (D3/12/46/24-28) dated in pencil June 6-7, 1955 with the title also in pencil “a proceeding which, by analogy, might be called “The Value Theory of Labour””; but the content of which is written in ink. Because of this disparity (date and title in pencil; content in ink) we conjecture that although the date and title were clearly written in June of 1955, the content (ink) portion was penned sometime in the 1940s; furthermore, for reasons we spell out directly, we conjecture that this date was in early 1943, due to the resonance this document has with documents from this period. From the “Value Theory of Labour” document, we find the interesting proposition that no-profit prices, that is to say, prices ascertained when the wage share is unity and the rate of profit zero, can be arrived at via two methods: what Sraffa calls the “direct method” which involves the reduction to quantities of dated labor, and the “indirect method” which involves setting the rate of profit equal to zero in the original system of equations (note that in the following quotation the deleted portions have been excluded so as to facilitate the flow of the content Sraffa retained; the footnote to this contains the entire passage, including the deletions):

“Discussions on the relations between labour + value are apt to concentrate on the influence of the former upon the latter. There is some interest however in looking into the opposite (wrong) other

¹⁰ We thank Gary Mongiovi who in a recent session (February 2009) at the Eastern Economics Association asked of the present writer clarification as to what Sraffa meant by Wicksell’s “puzzles being answered.” It was not until a subsequent visit to the Wren Library in July of 2010 that the appropriate documents were found by the present writer that better clarifies this question; but nonetheless the seed of that question remained ever since Professor Mongiovi planted it in the present author’s head.

end of the telescope, namely starting from the value to discover the quantity of labour. The quantity of labour contained in a product can be *formed* ^{ascertained} *directly*, when the ‘reduction’ of the commodity + of the materials used in its production to their ‘ultimate’ labour content can be carried out in a ‘finite number of moves’ it ^{at one remove} can be done by finding the limit of the sum of an infinite series, when the reduction has always a residue of commodities [a very different affair: when wages tend to zero]. In these cases it can be verified that the result of the direct method of ascertaining quantity of labour always agrees with the *indirect method*, that of solving the equations for values, i.e. after making $r = 0$: for it is clear that (on the usual assumption of uniform labour, or some equivalent assumption) that value of a commodity must be equal to the value (+ therefore proportional to the quantity) of labour which directly or indirectly ‘enters’ it – since all the proceeds go to labour. The indirect method however comes into its own in cases in which direct ascertainment becomes impossible ^{is not possible, at least in a way acceptable to common-sense} and indeed inconceivable, as in the case of joint products” (D3/12/46/24-25; italicized emphasis added).¹¹

Notice here that Sraffa actually identifies three different sets of prices, the first two of which represent alternative methods to arrive at the “value theory of labour” values, and the third “a very different affair”:

Value theory of Labour - Direct method : Reduction to dated quantities of labor

Value theory of Labour – Indirect method : Solve system of equations when $r = 0$ and $\omega^{12} = 1$

“...very different affair...”-prices : Solve system when $r = R$ and $\omega = 0$.

We find this conceptualization of prices conceived at the “opposite (wrong) end of the telescope” is very fascinating indeed. We begin to find here a very sophisticated approach to the price phenomenon. Legitimate economic and scientific merit results by the conceptualization of the price form at the opposite ends of the income distribution. At the upper end of the income distribution when the wage share is unity and the wage rate is complete there will “spring directly” a set of commodity prices and the wage rate that allow Sraffa to conceive of the fundamental constancy between the value of social capital and the value of the

¹¹ The full citation including the deletions is as follows:

“Discussions on the relations between ~~quantity of~~ labour + value are apt to ^{concentrate on} ~~concern~~ the influence of the former upon the latter. There is ~~much~~ some interest however ~~also~~ in looking ~~at the matter in the opposite direction and using~~ into the opposite (wrong) other end of the telescope, namely starting from the value to ~~find~~ discover the quantity of labour. The quantity of labour contained in a product can ~~in some cases~~ be formed ^{ascertained} *directly*, when the ~~by~~ ‘reduction’ of the commodity + of the materials used in its production to their ‘ultimate’ labour content ~~when the reduction~~ can be carried out ~~completed~~ in a finite number of moves’ it ^{at one remove} it can be done by finding the limit ~~when~~ of the sum of an infinite series, when the reduction has always a residue of commodities [a very different affair: when wages tend to zero]. In these cases it can be verified that the result of the direct method of ascertaining quantity of labour always agrees with the indirect method, that of solving the equations for values, i.e. after making $r = 0$: for it is clear that (on the usual assumption of uniform labour, or some equivalent assumption) that value of a commodity must be equal to the value (+ therefore proportional to the quantity) of labour which directly or indirectly ‘enters’ it – since all the proceeds go to labour. The indirect method however comes into its own in cases in which ~~the~~ direct ascertainment becomes impossible ^{is not possible, at least in a way acceptable to common-sense} and indeed inconceivable, as in the case of joint products” (D3/12/46/24-25).

¹² The symbol omega (ω) is used throughout this paper to denote the wage share; accordingly strictly speaking Pasinetti’s “complete wage rate” is defined as the wage rate (numeraire per unit labor) when the wage share is unity and the rate of profits is zero ($\omega = 1 : r = 0$); i.e. $w^{\text{complete}} = w^{\omega=1: r=0} = w^0$; similarly for “complete” prices of commodities (numeraire per unit commodity i); i.e. $p_i^{\text{complete}} = p_i^{\omega=1: r=0} = p_i^0, i = 1, \dots, n$. These latter prices are precisely the VTL-prices ascertained via the “indirect” method referenced in the text.

social output (R) in his economic system. To use a phrase that he repeatedly employed in the 1950s when he embarked on finally getting his inquiries into a publishable form, already here Sraffa is searching for a “rock to cling to”. Sraffa shows that these upper-limit prices can be “sprung” through two alternative but equivalent methods, the “direct method” through the reduction to quantities of dated labor, or the “indirect method” through solving the system of equations when the wage share is unity and the rate of profit is zero. Sraffa shows in notes from this time that the reduction methodology is perfectly determined at the unitary wage rate. The relationship in the reduction equation between the date-reduced living labor and the value of the “commodity residue” is very transparent and unequivocal when $\omega = 1$ and $r = 0$. Here the labor theory of value holds in that the prices determined are equivalent to direct and indirect labor embodied in the production of the commodity. And this is the essence of Sraffa’s sub-system. In *Production of Commodities* Sraffa links the relationship between the direct and indirect methods (reduction and $\omega = 1 : r = 0$) precisely in the Appendix A On ‘Sub-Systems’:

“Thus in a sub-system we see at a glance, as an aggregate, the same quantity of labour that we obtained as the sum of a series of terms when we trace back the successive stages of the production of the commodity (Ch. VI)” (Sraffa, 1960, p. 89).

The sub-system method allows Sraffa to conceive of the physical structure of the economic system in the absence of income distribution. Hence we contend that what in 1943 Sraffa calls the “indirect” method evolves into in the sub-systems approach he adopted in his book. A brilliant visual expression of this is given in Harcourt and Massaro (1964)’s important article on Sraffa’s subsystem (the diagrams have been reproduced in an Appendix to this paper).¹³ There the three-commodity world is conceived (fig. 1, p. 717) and is subsequently partitioned into a three-commodity integrated sub-system (fig. 2, p. 718). It is interesting to note that the rate of profit does not appear in these diagrams, only the physical heterogeneous means of production requirements per industry, the quantum of living labor per industry, and the physical gross output conceived as produced means of production requirements and the physical net product. In this system if the wage share is unity the prices correspond to the direct and indirect labor embodied.

These “very different affair”- prices correspond to the set of prices at the zero wage rate and the maximum rate of profit, what Pasinetti calls the “pure capital theory of value” (1977, pp. 78-80). Here we find very clearly that Sraffa as early as the 1940s begins to conceive of the dual extremes of the distribution problematic (with zero profits and pure wages at one pole and pure profits and zero wages at the other) as both giving rise to an economically-relevant structure of prices. This arises by Sraffa’s willingness to view the system through the two “opposite...end(s) of the telescope”. What will become important for Sraffa is that even if these prices are used, i.e. prices ascertained when the wage share is zero and profits are at their maximum, the ratio of fundamental constancy becomes no less evident and the maximum rate of profit is seen to “coincide” with the fundamental constancy of the “complete” labor theory of value price system. Sraffa tells us this in Ch. VI on the reduction:

“It is only at $r = R$ that the residue becomes all important as the sole determinant of the price of the product” (Sraffa, 1960, p. 35).

So here we have emergent for the case of single product industries a fundamental equivalence between the reduction equations and the solution to the set of prices when the wage share is unity and the rate of profit zero. The invariance hypothesis can now be postulated, and the economic system so-constructed and re-

¹³ We read there that “[t]he writers are grateful to Mr. Sraffa for his helpful comments on a draft of this note” (p. 715).

arranged and “unwanted parts chipped away” such that the commodity residue components in the sub-system are in the proportions necessary to establish “recurrence” and “balance” when date-reduced. This invariance also expresses itself at the set of prices when the wage share is zero and profits are at their maximum in the form of the maximum rate of profits. And because of the construct of the Standard system, what goes on in-between these extreme values as regards income distribution now becomes quite transparent indeed. Sraffa revisits this equivalence between the two approaches, the direct reduction approach and the indirect sub-systems approach, when extending the case to fixed capital. What he shows is that although the reduction methodology no longer holds, the sub-system approach *does*.¹⁴ And this leads Sraffa to conclude that his theory is robust. We will return to this in the following section when discussing the development of Sraffa’s equations from the 1940s.

The relevance this has for Wiskell’s “puzzles” relates to the problem of fixed capital (or a “durable consumption good”), specifically the dependence in this instance of the average period of investment on the rate of interest, and the resulting circular reasoning that the attributed to Böhm-Bawerk in this case. We read later in this note by Sraffa:

“[F]or Fixed Capital there is no necessary relation between the quantity of labour + the value which it sheds (passes on) in any one year. Consider a machine with a life of 10 years, which works with equal efficiency in each successive year. One tenth of the labour contained in it passes into the product of each year – according to the diminution of Value at $r = 0$. But at any level of $r > 0$ the amortization in the first year falls below $1/10$ of the original price + at sufficiently high levels of r will even fall below $1/10$ of the wages of labour contained in the machine. The repugnance to accept this is no doubt at the root of the belief that with the rise of r the ‘transfer of “labour contained in the fixed capital” to the product’ is delayed to the later years of the life of the asset; thus confusing the transfer of price at the ruling rate of profit, which is regulated by the amortization quotas appropriate to that level of r , with the transfer of labour which is determined proportional to the value at $r = 0$. (This is the case of Wicksell, although he expresses it as a lengthening of the period of production: this however is equivalent to a delay in the transfer of labour to the product)¹

¹ See Lectures I 184 where it is said that in the case of capital goods ‘the average investment period will be dependent on the rate of interest, even with simple interest’ (D3/12/46/27-28).¹⁵

¹⁴ Two important paragraphs in Chapter IX Other Effects of Joint Production speak directly to this. In opening paragraph 66 Sraffa notes that the expressed intent of this chapter is: “to see to what extent other conclusions reached in the case of single product industries are applicable to joint products”. He notes “the rule that, when the rate of profits is zero, the relative value of commodities is proportional to the quantity of labour which, directly and indirectly, has gone to produce them” no longer holds for joint products. He comments that:

“we certainly get no help from the ‘Reduction’ approach...for this method seems totally inapplicable to the case of joint-products...However, with the system of single-product industries we had an alternative if less intuitive {i.e. “indirect” – SC} approach in the method of ‘Sub-systems’...” (Sraffa, 1960, p. 56).

He returns to this idea two section later in section 68:

“As noted above, while the method just outlined {i.e. the valid incorporation of joint-products in the theory - SC} is an extension of the approach by sub-systems, there is no equivalent in the case of joint-products to the alternative method, namely Reduction to a series of dated labour terms. In effect it is of the essence of such a Reduction that each commodity should be produced separately and by only one industry, and the whole operation consists in tracing back the successive stages of a single-track productive process” (Sraffa 1960, p. 58).

¹⁵ The relevant passage from Wicksell is as follows:

“It should be pointed out here that the assumption that the average period of investment is independent of the rate of interest (i.e. of simple interest) only applies, strictly speaking, where several different capital investments relate to one and the same future act of consumption (as in Böhm-Bawerk’s example). In the opposite case,

To establish our conjecture that the above text was written in early 1943, consider a very similar documented penned 13 January 1943:

D3/12/33/83 (1)

“13.1.43

Quantity of Labour and Value (Define)

In the original scheme I have two independent ways for measuring the q. of lab. contained in a commodity: 1) “tracing back” and finding the quant. of lab. used directly and indirectly (i.e Reduction), and 2) solving the equation of $r = 0$.

Consequently I can say that the q. of labour (measured according to 1) determines value (measure acc. to 2).” (D3/12/33/83 : 1)

And we also find the problems this approach has for joint production and fixed capital, and once again Sraffa revisits Wicksell, this time related specifically to the latter’s approach to fixed capital and Åckerman’s problem:

“When we come to the next extensioning, to fixed capital and to joint products, a greater effort of abstraction is required. There is however a difference between the two cases: for fixed capital, some people may think that there is no insuperable (great) difficulty in accepting the idea that the quantity of labour that passes into the product of a machine lasting n years with constant efficiency is $\frac{1}{n}$ of that originally contained in the machine. While for joint products no intuitive support is available. However, Wicksell finds an “insuperable” difficulty in applying the principle of “previously done labour” (Lect. I, 260 in the case of fixed capital. “Clearly¹⁶...)

Having gone so far, we now extend the definition to all cases, including those in which the “ordinary notion” cannot be applied accurately, or not even imagined: the quantity of labour contained in a commodity is proportional to the roots of the equations of production for $r = 0$, in other words, to its value” (D3/12/33/83 : 4).

where one (or more) factors of production are invested in a single capital-good or durable consumption good, it may easily be seen that the average investment-period will be dependent on the rate of interest, even with simple interest” (Wicksell, 1934, p. 184).

¹⁶ Sraffa leaves the Wicksell quote simply as “Clearly...”. When we go to the copy of Wickell’s *Lectures on Political Economy*, Volume I, that is in Sraffa’s library (Sraffa 2095), we find the following passage heavily marked; note all underlines are in Sraffa’s hand, as are the lines at the margin:

“But if the machine has been in use for over a year or several years there remains only one part of the “annual use”, which, for the sake of simplicity, is assumed by the author to be constant in size of technical value. Clearly it is then quite impossible to decide how much of the previously invested labour recourses remain “stored up” in the capital-good. In fact the question has no meaning to which any proper sense can be attached. For the annual uses successively following one another constitute a kind of joint-supply (to adopt Marshalls’ terminology) and fundamentally it is just as absurd to ask how much labour is invested in either one or the other annual use as to try to find out what part of a pasture goes into wool and what part into mutton. It is only at the margin of production that these quantities can be differentiated and have a concrete significance assigned to them.” (Wicksell, 1934, pg. 260).

Thus the “value theory of labour”, or prices ascertained when the wage share is unity, helps to answer certain puzzles in Wicksell, which we find in the latter’s approach to the problems inherent in durable-capital models. We leave the question of fixed capital aside for now and will return to it in the development of Sraffa’s equations in the period 1942-1944; especially of interest regarding fixed capital is the realization through laborious effort that the reduction is not valid in the case of fixed capital. What is important here is the fact that for Sraffa the reduction to dated quantities of labor in the (valid) case of strictly circulating capital represents an alternative yet equivalent methodology for determination of the set of prices ascertained when the wage share is unity. And these “value theory of labour” or VTL-prices, whether determined “directly” via the reduction or “indirectly” via setting the rate of profit equal to zero, will play a very powerful role in Sraffa’s inquiries from the early 1940s through the 1950s up to the final drafts of his book. Sraffa would consistently and repeatedly use these prices in order to ascertain important economic quantities and economic ratios, including especially the constancy property associated with his Hypothesis. Indeed, we find that Sraffa “mixes the history” in the sense that he continually derives relations relevant for capitalistic-society proper (i.e. $\omega < 1 : 0 < r < R$) through utilization of the VTL-price structure “historically valid” for “early and rude” generalized commodity producing society.¹⁷

We now turn to the question of the constancy property itself. Kurz and Salvadori (2010) note that since as early as the first period of Sraffa’s constructive endeavors he had been aware of the fundamental constancy of social capital with respect to social product:

“[I]n a paper he had begun to write in February 1931 he contemplated the case in which ‘the value of total capital in terms of total goods produced cannot vary (as income distribution changes), since the goods are composed exactly in the same proportions as the capitals which have produced them’ (D3/12/7 : 153 (3))” (Kurz and Salvadori, 2010, p. 263).

Sraffa would call such a system where “item by item” the input side of the equation would consist of the same goods in the same proportions as the output side of the equation a “repetitive system”, and an economic system that does not have this characteristic a “non-repetitive system”. And it was the “repetitive” nature of the economic system that Sraffa was keen on developing. Certainly by August of 1942, in an important set of notes entitled “Crosscap”, Sraffa attributed a variant of this constancy to the manner of inquiry initiated by Marx (see also Gilibert 2003 and Bellofiore 2008, who make much of the same claim). We read there that:

“This manoeuvre is pivotal for the whole operation and everything depends on its success. We should proceed as follows. First, by developing the 1st equations {i.e. production for subsistence}, then the second (with r) {i.e. the original surplus equations}, then by introducing w as a variable. This is the sensitive point: we must tell everything, but must not reveal the secret about the constant ratio between C and $V+S$. We can possibly say that the organic composition (expressed in vulgar terms)

¹⁷ Now of course no such “early and rude” society ever really existed, and this is related to the question of the “historical” labor theory of value, a point made quite convincingly in Kurz and Salvadori (2010). They argue that beginning with Smith, and including the likes of Torrens and Engels, it “had been contended that the labour theory of value holds in ‘primitive’ societies”, but that “apparently Sraffa was not convinced” (Kurz and Salvadori, 2010, p. 198). They cite very interesting evidence that Sraffa seriously considered the merits of this question, studying “books on economic history, anthropology, and ethnology and noted *inter alia* passages dealing with the historical interpretation of the theory” (Id.). It is ultimately shown that Sraffa came down against the idea of a “historical” approach to the labor theory, and we concur with this assessment. For Sraffa the question turned primarily on the analytical distinctions between the two separate states of society.

of the two groups is identical. Perhaps, we may examine in detail the effects of a change in r or w on the price of individual commodities. In any case, better to leave the clue for a later time...so far the inquiry must have been scrupulously developed in vulgar terms, definitely avoiding any reference to similarity to Old Moor {Marx} and particularly the use of fundamental notions” (D3/12/16/17).¹⁸

Sraffa conceived of a constancy which during this period expresses itself in at least two alternative (and compatible) ways: that of the constancy of the capital stock to the gross product; and that of the constancy of the net product to the capital stock. In our reading of Sraffa we see him move between these two conceptions sometimes as if they were the same, with both being denoted by the capital letter R . He would eventually settle on the latter expression, namely that R = net product divided by means of production, but that he also conceived of R as the capital-gross output ratio can be seen in two ways. First consider notes Sraffa wrote on June 6, 1943 regarding Cassel (1935) *On Quantitative Thinking in Economics* and how he (Cassel) approvingly cites an important article on Austrian reduction methodology by Jacob Marschak (1934):

“p. 23

{Cassel} Approves of Marschak’s (E.J. March 1935)¹⁹ definition of the period of production as inverse of way R . [N.B. Marschak defines it as a ratio of values (meaning ‘prices’) without restriction; *but does not mention Hypothesis, which is essential for it to be true!*]” (D1/91/43; emphasis added).

¹⁸ The original is written in Italian. I rely on Bellofiore’s (2008) translation to English (Bellofiore 2008, p. 90, n. 18).

¹⁹ There are two mistakes related to this quote. The first is a mistake in Cassel (1935), where he (Cassel) erroneously attributes to Marshall what in fact is Marschak’s article, “A Note on the Period of Production”, *Economic Journal*, 1934. The second mistake is in Sraffa’s note, where he mistakenly cites 1935 for Marschak’s article when in fact the correct year is 1934. Marschak’s article is a review of two important articles on the average period of production, both of which appeared in the December 1933 issue of the *Economic Journal*; Martin Hill’s “The period of production and industrial fluctuations” and C.H.P. Gifford’s “The concept of the length of the period of production”. Neither of these latter articles appears in the notes that we have amassed from the Sraffa Papers, and the present author did not inquire whether they appear in Sraffa’s Library on our last visit to the Wren. However, Sraffa does cite on three occasions in our transcriptions an important article by Rangar Nurkse (1935, “The schematic representation of the structure of production”, *Review of Economic Studies*, vol. 2). In that article Nurkse approvingly cites Martin Hill’s article especially along the lines of the latter’s approach to the period of production from the “forward-looking aspect...considering the future dates at which the products of current services of the *present* stock of original factors mature in consumable form” (Nurkse, 1935, p. 233; emphasis in text). Sraffa too will consider a similar notion of “forward-looking”, and will refer to this as “compounding”; “This operation we call compounding: it is the inverse of Reduction” (D3/12/26/4). On Sraffa on Nurkse see D1/91/61, where Nurkse is placed (among others) under the heading of “Capital Sottisier”, where “sottisier” is translated as “a collection of howlers” (*Harrap’s Unabridged French-English Dictionary*, p. 1141); hence we can assume that “Capital Sottisier” can mean “serious errors in the theory of capital” or something to that effect; see also D1/91/63 and D3/12/29/8 (5), where Nurkse is identified as a “follower of Böhm-Bawerk”:

“[S]ince Burchardt in 1939 made accessible to marg. {marginalist} economists some of the ideas of Marx on the circulation of commod. by summarizing them and comparing them with BB’s {Böhm-Bawerk} (first hand reading being taboo – except at that time in Germany, where Hitler promptly arose to stop it) both critics and followers of BB have freely admitted that that is, at least the more general case (Nurkse, Fleming, Lindahl).” (D3/12/29/8 : 5)

Reference is to Fritz Adolf Burchardt (1939) *Die schemata des stationären Krieslaufs bei Böhm-Bawerk und Marx* (Sraffa 4708). The Fleming reference is to J. Marcus Fleming (1935) “The period of production and derived concepts”, *Review of Economic Studies*; Sraffa cites the Fleming article in D1/91/63, D3/12/29 (8), and D3/12/29/25. The Lindahl reference is Erik Lindahl’s (1939) *Studies in the theory of money and capital*. The Nurkse (1935) article cited is actually a very readable and thorough account of the two departmental schemes of reproduction in Marx within the Austrian date-reduction framework and includes a very interesting schematic diagram of the integrated economy (p. 235). Nurkse makes the following very interesting parallel of Marx to Böhm-Bawerk:

“It is curious to reflect how much better in some respects the triangular picture of the productive process – with its emphasis on labour as the ‘original factor of production’ – would have suited Marx. With the fundamental idea of labour as the ‘source of all wealth,’ the labor theory of value (‘modernised’ by Marx) and

In turning to the quote in Cassel (1935), which is Sraffa's copy is heavily marked, we find significant relevance for Sraffa's Hypothesis from page 22 (not cited in Sraffa's note) and page 23 (which is cited in Sraffa's note). The relevant passage, quoted in full, is the following:

“Before entering farther upon this subject...we ought to observe that a more natural and reliable measure of the importance of capital in the social economy could be obtained simply by calculating the *quotient between the capital and the income of the economy*. If we express both capital and income in terms of money, and if we assume a state of equilibrium in which all prices are fixed and where therefore real capital has a price corresponding to its cost of production, this quotient has a definite quantitative meaning, and is obviously an important characteristic of the social economy.

It is interesting to note that efforts to find a satisfactory measure of the average period of production have led to the adoption of that period which, as Mr. {Marschak} says in the *Economic Journal* (March 1934), is “identical with another interesting economic magnitude, viz, the ratio of the total value of existing commodities (“stocks”) to the value of the current income or consumption (“flow”)”. If this is so, we have all the more reason for abandoning the whole concept of an ‘average period of production’ and for concentrating our attention instead on the clear and measurable concept of the *quotient between capital and income*” (Cassel, 1935, pp. 22-23; emphasis added).²⁰

Here we find in Cassel's “quotient” a very similar notion to Sraffa's constancy Hypothesis. As late as 1945 Sraffa had in mind the idea of the ratio R consisted of the ratio of capital to gross output – Cassel's quotient - *not* net product to total capital as would be the case if R was equivalent to the maximum rate of profit. This is seen in notes written on Bortkiewicz in 1943 that Sraffa revisited on 21 December 1945, in an interesting numeric example that shows unambiguously this claim (i.e. that R here is Cassel's quotient, not the maximum rate of profit):

“How {Bortkiewicz's} statement is false in general can be seen from the example of wheat. Suppose that to produce 10 bushels, there are required 2 bushels of seed and 8 days of labour: so that the

the wage-fund doctrine (“modernised” by Böhm-Bawerk) both flow from a common stock of conceptions and misconceptions of classical economics” (Nurkse, 1935, p. 238, note 1).

Rangar Nurkse, and Estonian émigré to Canada, was mostly known for his work on balanced and unbalanced growth in the realm of international trade theory as well as his work for the League of Nations; he suffered an untimely death in May 1959 at the young age of 52, at the height of his career. For a brief biographical essay on Nurkse see Gottfried Haberler's Introduction to *Equilibrium and Growth in the World Economy: Economic Essays by Rangar Nurkse*, Harvard Economic Studies, Volume CXVIII (1962).

²⁰ It is of interest to consider Cassel's own opinion of his present work cited and the disdain he had for subjective elements in the theory of value:

“A thorough reconstruction of economics as a quantitative science is urgently needed and, indeed, long overdue. In no other branch of modern science would such a general lack of quantitative definiteness as still prevails in economics be tolerated. The aim of the reconstruction must be to present the actual facts and problems of economic life in the most distinct form and, as far as possible, in measurable terms. To succeed in this work, economists must relieve themselves of the oppressive burden of withered notions and barren dogmas, inherited from highly theoretical and scholastic controversies of a past century, and face with youthful vigour and up-to-date scientific equipment the problems of their own age” (Cassel, 1935, p. *v*).

Wesley Mitchell in *Lecture Notes on Types of Economic Theory*, Vol. II, notes that Cassel, who studied as an engineer, had a “disinclination to accept what he refers to...as ‘the subjective theory of value’” which was “natural enough in an engineer” (Mitchell, 1969, p. 423).

product of 1 days labour is one bushel. Now suppose an invention, by which to produce 10 bushels there are required 1 bushel as seed and 8 days labour; so that the product of each day's labour is $1 \frac{1}{8}$ bushel. Obviously the increased productivity of labour is the same in all stages (in producing the seed, and in producing the final wheat) and yet the period of production is changed.

[My example involves a fall in the Org. Composition, i.e. a rise {this is a typo by Sraffa; clearly it should read "fall"} of R from $\frac{1}{5}$ to $\frac{1}{10}$. This ought to be reversed, which can be done by assuming that the invention, or "other" method, requires $2 \frac{1}{2}$ bushels and only 5 days labour to produce 10 bushels; so that the product per day becomes $1 \frac{1}{2}$ bushels and $R = \frac{1}{4}$ after the invention.]”
(D1/91/26 on back)

To see that Sraffa here refers to R as the ratio of means of production to gross output, consider the following tabular representation of Sraffa's numeric example. In this quote Sraffa actually considers an original method of production, and two distinct scenarios of technical change through two distinct "inventions". The first case of "invention" involves a fall in both the organic composition of capital (OCK) and in the Casselian quotient R coupled with a rise in productivity of labor. The second "reversed" case of "invention" involves an across the board rise in the OCK, the Casselian R, and in labor productivity.

Let us call the original technique of production technique α , that associated with a fall in the OCK and the Casselian R technique β , and that associated with an across the board rise in all ratios technique γ . As with Sraffa's example, each is a one-commodity wheat producing technique, the inputs of which are wheat-as-seed, denoted A_{wh} , and living labor, denoted L_{wh} , which at the end of the harvest produces a gross output of wheat, denoted Q_{wh} . The physical net product is denoted Y_{wh} and is the difference between the gross output and wheat-as-seed input requirements, i.e. $Y_{wh} = Q_{wh} - A_{wh}$. Wheat is measured in physical units of bushels (bu.) and labor is measured in physical units of hours per day (day). The three relevant economic ratios in this simple system are the OCK, defined as the means of production divided by the labor requirements, the Casselian R, defined as the means of production divided by the gross output, and the productivity of labor, defined as the net product divided by the labor requirements. Thus:

$$A_{wh}^i = \text{wheat-as-seed input requirements for technique } i = bu.$$

$$L_{wh}^i = \text{living labor input requirements for technique } i = day$$

$$Q_{wh}^i = \text{wheat-as-output for technique } i = bu.$$

$$Y_{wh}^i = \text{wheat-as-net product for technique } i = Q_{wh}^i - A_{wh}^i = bu.$$

$$OCK_{wh}^i = \frac{A_{wh}^i}{L_{wh}^i} = \text{organic composition of capital for technique } i = \frac{bu.}{day}$$

$$y_{wh}^i = \frac{Y_{wh}^i}{L_{wh}^i} = \text{productivity of labor for technique } i = \frac{bu.}{day}$$

$$R_{wh}^i = \frac{A_{wh}^i}{Q_{wh}^i} = \text{Casselien quotient R for technique } i = scalar$$

Table 1: Sraffa's Numeric example of Wheat production across three techniques from D3/91/26

Technique	Round of production	Net Product $Y = Q - A$	OCK = A/L	$y = Y/L$	$R = A/Q$
Technique α	$[A_{wh}^{\alpha} = 2bu. \oplus L_{wh}^{\alpha} = 8 \text{ day}] \rightarrow Q_{wh}^{\alpha} = 10bu.$	$Y_{wh}^{\alpha} = 8bu.$	$OCK_{wh}^{\alpha} = \frac{2bu.}{8 \text{ day}} = \frac{1 \text{ bu.}}{4 \text{ day}}$	$y_{wh}^{\alpha} = \frac{8bu.}{8 \text{ day}} = 1 \frac{bu.}{\text{day}}$	$R_{wh}^{\alpha} = \frac{2bu.}{10bu.} = \frac{1}{5}$
Technique β	$[A_{wh}^{\beta} = 1bu. \oplus L_{wh}^{\beta} = 8 \text{ day}] \rightarrow Q_{wh}^{\beta} = 10bu.$	$Y_{wh}^{\beta} = 9bu.$	$OCK_{wh}^{\beta} = \frac{1bu.}{8 \text{ day}} = \frac{1 \text{ bu.}}{8 \text{ day}}$	$y_{wh}^{\beta} = \frac{9bu.}{8 \text{ day}} = 1 \frac{1 \text{ bu.}}{8 \text{ day}}$	$R_{wh}^{\beta} = \frac{1bu.}{10bu.} = \frac{1}{10}$
Technique γ	$[A_{wh}^{\gamma} = 2\frac{1}{2}bu. \oplus L_{wh}^{\gamma} = 5 \text{ day}] \rightarrow Q_{wh}^{\gamma} = 10bu.$	$Y_{wh}^{\gamma} = 7\frac{1}{2}bu.$	$OCK_{wh}^{\gamma} = \frac{2.5bu.}{5 \text{ day}} = \frac{1 \text{ bu.}}{2 \text{ day}}$	$y_{wh}^{\gamma} = \frac{7.5bu.}{5 \text{ day}} = 1 \frac{1 \text{ bu.}}{2 \text{ day}}$	$R_{wh}^{\gamma} = \frac{2.5bu.}{10bu.} = \frac{1}{4}$

Table 1 demonstrates that, at least for Sraffa's notes penned in D1/91/26 dated December 21, 1945, the ratio R in fact amounts to the Casselian quotient. As further evidence of the notion that at times the ratio R represents the Casselian quotient, Sraffa in many places distinguishes "R" from "max r". One such example appears in the following, dated in the important set of notes entitled "Hypothesis" from January 27, 1944

"The original Hypotheses (that the commod. on the right and left are in constant ratio) and the derived linear relation between r, w, and max r may not be two identical propositions as I have thought so far. The second (which is what really matters to me) may be true when the first is not. The first may be a sufficient but not necessary condition for the second" (D3/12/36/63)

There are more instances of Sraffa's clear distinction between "R" and "max r". For example, in D3/12/21/64-65 written September 1942 the maximum rate of profit is expressed in terms of an early effort at the reduction as:

$$\text{"Maximum } \underline{r} \text{ (i.e. for } w = 0) \frac{cr}{c(1+r)^n - c} = \frac{r}{(1+r)^n - 1} = k \text{" (D3/12/21/64)}$$

Whereas later in the document an alternative expression is given for R:

"We obtain the equation (with K = capital stock) P = product

$$(c + v) \frac{(1+r)^n - 1}{r} = c + v + Kr = P \text{ also } CR = P \text{ (R = constant)" (D3/12/21/65).}$$

Clearly from the context here the Casselian quotient is represented by R.. However, at other times Sraffa does indeed refer to R as the maximum rate of profit. But at this juncture of Sraffa's analysis, the distinction

between R defined as the Casselian quotient vs. R defined as the maximum rate of profit is of relatively minor importance. Certainly the two are monotonically related such that the Casselian quotient is equal to one over one plus the maximum rate of profit, seen in terms of our notation above simply as:

$$R^{\text{Cassel}} = \frac{A}{Q} = \frac{A}{A+Y} = \frac{\frac{A}{A}}{\frac{A}{A} + \frac{Y}{A}} = \frac{1}{1 + R^{\text{max } r}}$$

Where here $R^{\text{max } r}$ = maximum rate of profit. What is of importance is the notion that, irrespective of which version of R considered, Sraffa's Hypothesis held fast to the idea that the ratio would remain constant in the face of changes in distribution. Sraffa actually writes two sets of notes that he entitled "Hypothesis" written almost one year removed. The first set was an eleven page document written on February 17, 1943 archived as D3/12/33/80/1-11. The other set of notes with same title (and the one referred to in much of the recent literature; CITE) was written on January 27, 1944 and is a 24 page document archived as D3/12/36/61-85. The latter document is especially of importance for the present essay, because it would be only two weeks later that Sraffa would "correct" Marx's organic composition of capital concept so that it would now resonate with the constancy Hypothesis that he had been intensely developing and using the previous two years. As we shall see, the role of the Reduction plays in both sets versions of his Hypothesis looms very large. The constancy of capital stock to output (whether net or gross) provided the anchor for the development of the various systems of equations he developed in the period prior to February 1944. It is to this latter task that we now turn.

II. The development of Sraffa's equations in the 1940s

The recent literature cited in the introductory portion of this essay develops Sraffa's equations primarily as they appear in the first phase of constructive activity (see also Garegnani and the "Turning Point"), known as his "first" and "second" equations (first = subsistence; second = surplus with Ramsey's influence). In this section we consider in some detail the evolution of Sraffa's equations in this important second phase of the 1940s, what have been called his "third equations". As we saw in the previous section, underlying the development of Sraffa's equations was adherence to his constancy Hypothesis; accordingly the idea of constancy in the ratio of social capital to social product provides the unifying theme in Sraffa's development of his system of equations in this period. As Sraffa developed his equations in this period, through his Hypothesis of constancy in the capital stock in the face of changes in income distribution (whether the variant of Cassel's quotient or the maximum rate of profit variant), he at the same time begins to conceive of the collection of equations in terms of an underlying structural adherence, the economic implications of which Sraffa deems as profound. And this underlying structural adherence of the system of equations begins early on for Sraffa to gain an independent-like existence, serves as the basis of the constancy property that he so insightfully Hypothesized the economic system to have, and will in the different expressions it gained in this period always serve the role of *numeraire* of the price system.

The development of this structural adherence early in the period beginning in Summer 1942 first expresses itself simply as the "Average Commodity", and then later in 1943 moves to what he will call the "All-Commodity". He will toy with the idea of the All-Commodity at an aggregate level and then, via the reduction equations, begins to conceive of an individual representative commodity he calls "Any commodity B". From here he introduces the notion of a commodity "made of itself", and later of a commodity

“constant made of itself”. Already we can begin to discern some of the fundamental properties of what would eventually emerge as the Standard system.

We begin with the manner Sraffa conceives of his equation system in August of 1942. On 31 August, just ten days after he penned his important notes “Crosscap” (D3/12/16/17), Sraffa writes out in detail his system of surplus-producing equations for what he would subsequently label the “discontinuous” case of point-input point-output production, equations he called “Agricultural Schemes”. The system of equations is reproduced in full as follows:

D3/12/19/1(1)

“31.8.42

A, B, C, ... are quantities of commodities

p_b, p_c, \dots their prices in terms of A (arbitrarily chosen)

Λ quantities of labour

Capital letters are constants, small letters variable

Agricultural community – period of turnover 1 year

seed, raw materials, tools wages profits product

⏟ ⏟ ⏟ ⏟

$$(A_a + p_b B_a + p_c C_a + \dots + p_n N_a + w \Lambda_a)(1+r) = A_t$$

$$(A_b + p_b B_b + p_c C_b + \dots + p_n N_b + w \Lambda_b)(1+r) = p_b B$$

⋮

$$(A_n + p_b B_n + p_c C_n + \dots + p_n N_n + w \Lambda_n)(1+r) = p_n N_t$$

$$A_a \geq 0$$

$$\sum_a^n A \leq A_t$$

$$\sum_a^n B \leq B_t$$

There are n equations,

n+1 variables ($n+1$, p's, w and r)

For any given value of w all the other variables can be found.

The commodities produced (represented by the right hand sides) are allotted in these ways: 1st, to replace the commodities $A_a, B_a, \dots, A_b, \dots$ etc. used up in production as raw materials, fuels, etc. (these form the constant part of capital, as their quantity does not change when the proportions in which the rest of the product is distributed between wages and profits are changed); 2nd to wages (the variable part of capital); 3rd to profits (the surplus left over when all capital has been replaced).

All the commodities produced can therefore be divided into three classes, according to the way in which they are disposed of. Many will be subdivided one part going to one class, and the other parts to each of the other classes.”

In Sraffa's Agricultural Schemes wage are paid *ante factum* and an arbitrary commodity (here commodity A) is set as the *numeraire*. The cycle of production is the annual harvest and as indicated the structure of this system of equations is of the point-input point point-outut variety. At this stage of the game, Sraffa recognizes the "three classes" that the commodities can be divided into, namely (1) means of production, (2) means of workers' consumption, and (3) means of capitalist consumption and/or investment. Notice here that Sraffa is clear to make the point that this tripartite division is strictly along the lines of how the commodities "are disposed of". Sraffa will here make an important distinction between the "disposal" of commodities vs. their production, and later in this note he remarks that for his purposes, it is the *production* of commodities that becomes more relevant:

"If we look at this crowd of commodities from the point of view of the way in which they have been produced, we notice great differences among the individuals composing the crowd. Accordingly, the proceeds of their sale will go in different proportions, on the one hand, to replace the constant and the variable capital; and again in different proportions to replace the variable capital and to the surplus. Similarly when the proportions of wages and profits are changed, their prices will vary in opposite directions, some rising and some falling, and to various extents. [They can be classified accordingly]

If however we look at these same commodities from the point of view of their disposal, we find that this choice is entirely determined by their fitness respectively as raw materials or tools of production, or as objects of consumption of the workers or the capitalists. The allotment of a commodity to one or the other class will have no connection with way in which it was produced: agriculture products may go to constant capital as raw materials, or to variable capital or surplus as foods; coal may equally go to all of these classes; and (to anticipate from the next section, on industrial production) the most elaborate products of the engineering industry may go indiscriminately to production or consumption. *Therefore, the classification by destination is totally independent from that of composition of their cost, and can be regarded as random of the latter point of view*" (D3/12/19/1 : 2; italicized emphasis added)

We find here that for Sraffa, as regards the costs of producing commodities, the particular disposal especially of the net product is immaterial. And it is here he begins to move away from the Marxian distinction with respect to the division of the net product. As is well known, Marx, and to the letter all followers of Marx, conceive of the net product (or value-added) as in the first instance already divided up into the "necessary" component and the "surplus" component. However we find that beginning here, Sraffa starts on a certain level to jettison the importance in this distinction and instead focuses on the net product (value-added) *as a whole*. And it is this aspect of the net product, namely that it consists of the unity of both the necessary and the surplus elements, that allows Sraffa to in his mind correctly conceive of the constancy ratio that characterizes his Hypothesis. This we see in the passage immediately following the one cited above:

"But if that is so, the relative value of the commodities making up each group, each group taken as an aggregate, will not vary in relation to the other groups, when the distribution between wages and profits changes.

Thus the total value of the commodities composing constant capital will not vary in relation to the rest of the product when the division of the latter varies. And an average unit of commodities going to wages will not vary in relation to an average unit of commodities going to profits varies {sic}.

Therefore, whatever the value we choose to give to wages (w), and whatever is the consequent value of the rate of profit (r), the relative values of these average units will remain unchanged” (D3/12/19/1 : 2-3; underlined emphasis Sraffa’s).

It is here that we begin to find in Sraffa the idea of invariance in “average” units of commodities; the seed (we argue) is thus planted for the eventual blossom known as the Standard system. Before moving on to the next system of equations Sraffa develops in the 1940s, namely what he will call “Industrial Schemes”, a quick word on how Sraffa in the “Agricultural” model begins to “mix the history” by conceiving of the of the aforementioned constancy first with VTL-prices (here determined “indirectly” by setting $r = 0$) and only later replacing the system with “the true r ” and hence the “true” prices of production:

“Now suppose that we want to find out what are the prices (p ’s) and the rate of profits for consumption ($V + S$). We can, in the equations give any arbitrary value to r and it happens to be convenient, for simplicity of calculation, to make $r = 0$. Wages will be equal to 1, or the total available for consumption. The equations will now be linear, and we get readily the values of the p ’s: these are purely arbitrary values, but they are as good as any others to estimate the relative value of (raw materials, etc.) = (constant capital) and consumption commodities; they are also good to estimate the total value of wages which we know only as a collection of commodities when they are $1/3$ of the available total, and of the $2/3$ that go to profits. Having the values of these three quantities C , V , and S , we have immediately that $r = S/(C+V)$.[□] This is no longer an arbitrary value, but the true r which we want.

$$□ \text{ or } (C + V)(1 + r) = C + V + S$$

We can now replace r by its known value in the equations and find the values of the p ’s; these too are the true values, and no longer arbitrary ones” (D3/12/19/1 : 3).

Notice here that Sraffa engages in what we now know as an imperfect method of valuation; he ascertains the values of the different components in the economic system according to $r = 0$, or what above we conceive as “indirectly-determined” VTL-prices. This allows him to (1) posit the constancy criterion with respect to R (whether the Casselian R or the maximum rate of profit variant), and (2) posit the unequivocal distribution between the necessary (wage) component vs. the surplus (profit) component. This methodology will remain a mainstay for Sraffa from this point onward.

Before moving into Sraffa’s Industrial Schemes, let us pause first to consider an important set of notes that he penned concurrent with this second set of equation, archived in D3/12/17, the folder of which he entitled “Prove and Finding Lists”. As we show in Carter (2009, 2010), contained in this folder are two vitally important sets of notes that Sraffa references in the March 1955 Majorca Draft, which as noted above was an important first draft of Part I of his book on single product industries. The relevant notes for our purposes are those drafted in September and November 1942 entitled “Exploitation” (D3/12/17/2-8). Here we find an important diagram that Sraffa would throughout his subsequent notes refer to as “spectrogram” (see D3/12/17/2). This diagram basically shows the “classification accordingly” of the different commodities with respect to their organic composition, and the subsequent rising and falling of prices as distribution changes, thus visualizing the idea that he wrote about in Agricultural Schemes. And it is in this set of notes that Sraffa first starts to speak about the “pool of profits” – i.e. the “social pool” that extracted unpaid labor was to be “deposited” and from which the different capitals would take their aliquot part according to their composition.

As indicated elsewhere we have made much of the development of the “profits pool” idea. For purposes of the present essay the importance of citing these notes is to reveal the importance of Sraffa’s intense study of the Austrian date-reduced approach especially regarding his close reading of Böhm-Bawerk. Regarding the process of exploitation as the source of the soon-to-be distributed profits, we read from a document dated 12 November, 1942:

Thus, the proceeds of a reduction of wages, don’t simply pass from the pockets of the workers to those of their employer. They go, as it were, into a social pool {insert in pencil “of profits”; possibly in 1955}, to which each industry contributed in proportion to the importance of its particular variable capital[□]; and which is then shared out, among the various industries, {insert in pencil “each receiving”} in proportion to their particular total capital (variable and constant).

{Note on back of page in pencil}

□ “Expand this and say ‘and its contribution has no relation whatever with the amount of [rather ‘unknown’] constant capital which it employs’ (Cp. Böhm-B., Cap. and Int. (Smart), p. 390” (D3/12/17/3)

When we go to the citation that Sraffa references in Böhm-Bawerk’s *Capital and Interest*, we find that it is taken from the chapter devoted to the exposition of Marx’s theory as compared to that of Rodbertus in the chapter on the “Exploitation Theory of Interest”:

“Another point to be noted is that, in business capital, Marx distinguishes two portions; of which one, in his peculiar terminology called Variable capital, is advanced for the wages of labour; the other, which he calls Constant capital, is advanced for the means of production. And Marx maintains that only the amount of variable capital has any influence on the quantity of surplus value obtainable, *the amount of the constant capital being in this respect of no account*” (Böhm-Bawerk, 1890, *Capital and Interest*; Smart translation; emphasis added).

We are of the opinion that Sraffa’s notes are a paraphrase of the italicized portion of the passage in Böhm-Bawerk. We further take this as further evidence that Sraffa was directly opposing the Austrian economist’s derision of Marx and his (Böhm-Bawerk’s) critique of the “exploitation theory”.

In September 1942 Sraffa begins to consider the more general and conceptually more sophisticated case of flow-input point-output processes of production, what he deems as “continuous production” that he labels “Industrial Schemes”, and collects these notes in a folder of the latter name archived as D3/12/21. In actual fact this folder contains what we can discern as four separate continuous documents written in the main in that month.²¹ The largest of the continuous documents is a 61 page set of notes without title but what we can call simply “Industrial Schemes” as it constitutes the lion’s share of the file folder. Another important document for purposes of the present essay is a two page document entitled “Measure of Organic Composition” written on 30 September 1942. This latter document is an extension of a single page written in

²¹ As is the case with much of the archival material, this large set of notes is scattered throughout the folder and accordingly is not archived in chronological order. This scattering reflects the state of the various folders that Sraffa left them in, an order (or better said a *disorder*) that each of the attempts at archiving retained (e.g. the Eatwell-Roncaglia “incomplete listing” of 1975; the Bharadwaj-Garegnani inventory of 1984; and finally the Smith Trinity Catalogue; see Smith 1998, pp. 43-45). To facilitate the chronological ordering of the 61 page document as well as that of D3/12/33 attached below is an Appendix that contains a Concordance the present author constructed during his time at the Archive).

the 61 page “Industrial Schemes” set of notes and paginated by Sraffa as “47 etc.” (archived as D3/12/21/3) of the same “Measure of OCK” title.²²

In moving with Sraffa from “Agricultural Schemes” to the development of the “Industrial schemes”, we find the Italian economist on September 6, 1942 posing the question in the following manner:

“We have so far dealt with production...with {a} single period of turnover... for all... branches of prod. and with the whole of the capital being entirely used up in the single period, so that nothing is left besides the product.

We now pass on to “continuous” production, i.e. different periods of turnover and “fixed capital”: the latter comprising those instruments the whole of which is necessary for the production of a single commodities, but which can be used over and over again” (D3/12/21/59)

Sraffa will in these equations fully embrace the reduction methodology of the Austrian approach, especially as regards the question of the period of production:

“The difficulties, acknowledged by Wicksell [? does he the 2d?] which are found in defining or measuring the period are two: 1st, that the period during which each unit of input (mat. or labour) takes to be transformed into product changes...with changes of \underline{r} ; 2nd that the average of such periods would change with \underline{r} , even if individually the periods were constant (owing to compound interest: explain $(1+r)^{n_1} + (1+r)^{n_2} \neq (1+r)^{\frac{n_1+n_2}{2}}$ or any other average n : the average changes with \underline{r} even if each n is constant – i.e. the average n that will make this equation true for all values of r).

It’s acknowledged by BB {Böhm-Bawerk}, W {Wicksell}, and everybody that \underline{n} is perfectly definite, unambiguous, and easily ascertained of $r = 0$. This is obvious for circ. cap., where each unit of input...preserves its identity can be traced down to the corresponding unit of output in which it is transformed” (D3/12/21/59).

Once again we find Sraffa utilizing the price form when the rate of profit is equal to zero, here conceived in terms of the average period of production (denoted “ n ”). The importance of this, as seen in the quote above, however, when the rate of profit becomes positive complications set in due precisely to the fact that the period of production changes with changes in the profit rate.

In the case of circulating capital, Sraffa notes that when the rate of profit is zero the “units of input” can be traced to the corresponding representation of the output; for fixed capital (again at the zero profit rate), there will be a rate of “amortization” that the value of the fixed capital will piecemeal pass to the output produced, defined as one over the number of years the machine lasts. Sraffa then continues:

“We now set up our equations for continuous production, with fixed capital, noticing:

1) some of the industries will be producing machines, ships, buildings, etc.

²² The other two independent documents are (1) a four page set of notes entitled “sum of value shed”, which reflects the problem of joint production and fixed capital (D3/12/21/9-6; note that this document is archived in reverse chronological order); and (2) a two page set of notes on excerpts from *Capital*, Volume III (which Sraffa read in German; see D3/12/21/79-80).

2) on the left hand side of the equations we still write the input that goes in during the course of a year [no longer ~~the~~ equal to the stocks at the beginning of the year: but also in previous equations it was the input of a year]: the stock of capital does not appear explicitly.

3) to the multiplier $(1 + r)$ of each industry we attach...the appropriate exponent n_a, n_b, n_c, \dots - that is to say the average time the units of input take before appearing in the product: this is a weighted arithmetical average - and (since the input is a number of heterogeneous commodities as well as quantities of labour) it is weighted by the values which the inputs have when $r = 0$ " (D3/12/21/60)

Here we begin to see the reduction methodology begin to manifest. To the profit factor $(1 + r)$ of each individual commodity input will be attached the appropriate exponents corresponding to the average period of its turnover, i.e. n_a, n_b, n_c, \dots . Sraffa paginates the above document (D3/12/21/60) as page 4; 24 pages later on Sraffa-paginated page 28 (archived as D3/12/21/50) appears the working form of the continuous production equations for circulating capital that Sraffa would subsequently refer for the remaining pages of these notes. There we find that Sraffa collects the different commodities according to the appropriate date. Written on or around 20 September 1942, Sraffa develops this system of equations as the following:

"Circulating and Fixed Capital

We distinguish units of input into two classes:

- a) Those which can be followed during their transit through production and traced to a given unit of output. These form Circ. Capital
- b) Those which cannot be so traced, because they are used in the production of a succession of units of output and only gradually wear out. These Fixed Cap.

Each unit of the first class has therefore a definite period \underline{n} of turnover, i.e. which characterises it; since a fresh unit is put in at every moment the number of units of each kind which are in transit at any time is proportional to that period; and between any time 0 and time \underline{n} , all the units in transit are replaced once; we call \underline{n} the period of turnover; \underline{n} is larger or smaller than 1 according as the period is $>$ $<$ 1 year.

Each unit of input is multiplied by its appropriate factor, $(1 + r)^n, (1 + r)^{n_2}, \dots$, where n_1, n_2, \dots are the periods of turnover belonging to each unit. We thus get:

$$\begin{aligned} (A_{a_1} + p_b B_{a_1} + \dots + p_n N_{a_1} + wL_{a_1})(1 + r)^{n_1} + (A_{a_2} + p_b B_{a_2} + \dots + p_n N_{a_2} + wL_{a_2})(1 + r)^{n_2} + \dots &= A_t \\ (A_{b_1} + p_b B_{b_1} + \dots + p_n N_{b_1} + wL_{b_1})(1 + r)^{n_1} + (A_{b_2} + p_b B_{b_2} + \dots + p_n N_{b_2} + wL_{b_2})(1 + r)^{n_2} + \dots &= p_b B_t \\ &\vdots \\ (A_{n_1} + p_b B_{n_1} + \dots + p_n N_{n_1} + wL_{n_1})(1 + r)^{n_1} + (A_{n_2} + p_b B_{n_2} + \dots + p_n N_{n_2} + wL_{n_2})(1 + r)^{n_2} &= p_n N_t \end{aligned}$$

To solve for any given value of \underline{w} we can proceed as before. We make $r = 0$ so that the factors $(1 + r)^n$ become $= 1$ and disappear. We then obtain arbitrary values of p_b, p_c, \dots which, with the given value of \underline{r} the true values of p_b, p_c, \dots are readily calculated" (D3/12/21/59-60).

It should be reiterated that the form of the equation system here represents the circulating capital model only; in subsequent notes Sraffa will make his attempt at fixed capital, and in doing so develop schedules of amortization where the value transferred to the product from the fixed capital input is expressed as a function of the profit rate according to the equation $p_m M \left(\frac{r}{(1+r)^n - 1} \right)$, where M = initial value of one machine (see D3/12/21/47). Sraffa refers to this as “the sum of value shed” or the “annual capital charge” and in D3/12/21/11 (Sraffa-pagination 45 quattro) he constructs the following table that relates the “accumulated sinking fund” to the “sum shed in a year”:

Table 2: “Annual Capital Charge (Sum Shed Annually by 1 Machine)” from D3/12/21/11

Thus end of	Accumulated Sinking Fund	One Year's interest on ditto (a)	Constant Annuity (b)	Sum shed in a year (a + b)
1 st year	$M \frac{r}{(1+r)^n - 1} = M \frac{r}{(1+r)^n - 1}$			$M \frac{r}{(1+r)^n - 1}$
2 nd year	$M \frac{r}{(1+r)^n - 1} \cdot \frac{(1+r)^2 - 1}{r} = M \frac{(1+r)^2 - 1}{(1+r)^n - 1}$	$M \frac{r(1+r) - r}{(1+r)^n - 1}$	$M \frac{r}{(1+r)^n - 1}$	$M \frac{r(1+r)}{(1+r)^n - 1}$
3 rd year ...	$M \frac{r}{(1+r)^n - 1} \cdot \frac{(1+r)^3 - 1}{r} = M \frac{(1+r)^3 - 1}{(1+r)^n - 1}$			$M \frac{r(1+r)^2}{(1+r)^n - 1}$
... m th year ...	$M \frac{r}{(1+r)^n - 1} \cdot \frac{(1+r)^m - 1}{r} = M \frac{(1+r)^m - 1}{(1+r)^n - 1}$			$M \frac{r(1+r)^{m-1}}{(1+r)^n - 1}$
... n th year	$M \frac{r}{(1+r)^n - 1} \cdot \frac{(1+r)^n - 1}{r} = M$			$M \frac{r(1+r)^{n-1}}{(1+r)^n - 1}$ The sum of this last column must be = M

At the bottom of this table Sraffa writes:

“[See Kent, Math. Pr. of Fin., p. 1078-9]

N.B. This represents merely the differences between the capital values of the machine in any two consecutive years” (D3/12/21/11)

The citation refers to Frederick Charles Kent (1927) *Mathematical Principles of Finance*, to which Sraffa owes the debt of construction of the table.²³

²³ In a review by E.B. Skinner that appears in 1928 in the *Bulletin of the American Mathematical Society* we read the following: “The real contribution of the book lies in the fine ten-place tables which have been worked out... The three principal functions of the mathematics of finance and their reciprocals are tabulated for thirty-six different rates for times to 300 years for rates from $\frac{1}{4}$ to $\frac{3}{4}$ per cent, to 200 years from rates from $\frac{7}{8}$ to $1\frac{1}{4}$ per cent and to 100 years for rate from $1\frac{3}{8}$ to $10\frac{1}{2}$ per cent. The auxiliary functions $(1+i)^{1/p}$, $j_{(p)}$, and $i/j_{(p)}$ for the same rates are given for $p = 2, 4, 6, 12$. The values of $(1+i)^n$ are given to fifteen decimal places for twelve fractional rates for one year intervals to 10 years, 10-year intervals from 10 to 100 and for 200 years. A few column sums taken at random indicate that the work is reasonably accurate. In one case the column sum

Sraffa would continue toying with the question of fixed capital using these amortization tables and formulae throughout these and other notes (see especially D3/12/24 – Notes on Reduction), but for our purposes what is of interest is the circulating capital form of the reduction equations that he developed in the Industrial Equations presented above.

Prior to the arrival of the above Industrial Equations, Sraffa makes the transition from the discontinuous Agricultural schemes using an interesting concept of what he refers to as “combines.” His model of “combines”, which he will return to and develop in a subsequent folder of October 1942 entitled “Closed Vertical Combines”, represents an attempt to “reduce” the input-process within the discontinuous model.²⁴ Sraffa starts this by considering two separate processes of integrated production of equal turnover, that of cotton spinning in the production of yarn used purely as an input to cotton-weaving in the production of cloth. Thus the output of the yarn-producing process is used wholesale as inputs for the cotton-weaving process the result of which is the finished (and hence consumable) commodity cloth. We read from Sraffa:

“Spinning and Weaving Combine

On the same data, we look upon some of them from a different point of view. Two of the ^{separate} branches of prod. considered were cotton spinning and cotton weaving; they had the same (unknown) period of turnover as all branches. We now regard them as a single branch: i.e. we assume that each spinning mill (or group of mills) is coupled with a weaving factory (or group of factories) the latter being such as to absorb the whole (and no more) of the yarn produced by the former. Suppose that in the system, commodity B was yarn and C was cloth. We had:

$$\begin{aligned} (A_b + \overline{p_b B_b} + p_c C_b + \dots + p_n N_b + wL_b)(1+r)^n &= \overline{p_b B_t} \\ (A_c + \overline{p_b B_c} + p_c C_c + \dots + p_n N_c + wL_c)(1+r)^n &= p_c C_t \end{aligned}$$

Was the general form: but since no yarn is used in the production of yarn the term $B_b = 0$ in the first equation and since the whole of yarn is used in the production of cloth, in the second the term $B_c = B_t$. Thus we write

$$\begin{aligned} (A_b + p_c C_b + \dots + p_n N_b + wL_b)(1+r)^n &= p_b B_t \\ (A_c + p_b B_t + p_c C_c + \dots + p_n N_c + wL_c)(1+r)^n &= p_c C_t \end{aligned}$$

differed from the computed value by 45 tenth place units but all other cases the agreement was much closer.

These admirably arranged tables will form a useful addition to existing tables...”(Skinner, 1928, p. 778).

²⁴ Sraffa would also return to the question of the production of the combine for fixed capital in an important set of documents written in November of 1942 archived as D3/12/26 entitled “Notes on value of stock, industrial equations, rotation and frequency, and reduction and compounds”. See D3/12/26/3-4 and Sraffa’s equation for the production process of “ships” where a two-period rotated fixed capital “ship-building” model is expressed in a one-year turnover equation.

If we regard the two industries as a single one, yarn (commodity B) disappears from the list of commodities and becomes simply an internal intermediate form: it equally disappears from input in the second equation and is replaced by the former B industry, thus:

$$\left[(A_b + p_c C_b + \dots + p_n N_b + wL_b)(1+r)^n + A_c + \dots + p_n N_c + wL_c \right] (1+r)^n = p_c C_t$$

or

$$(A_b + p_c C_b + \dots + p_n N_b + wL_b)(1+r)^{2n} + (A_c + \dots + p_n N_c + wL_c)(1+r)^n = p_c C_t''$$

But we know that the two original industries has the same period of turnover (n). We therefore know that their capital stocks varied, with changes in \underline{r} , merely in proportion to their respective (c + v)'s. The capital stock of the combined industry must, for any value of \underline{r} be equal to the sum of the capital stocks of the two separate sections.

But we know that, for $r = 0$ the capitals of the separate branches were,

$$(c_b + v_b) \frac{n^2}{2} \quad \text{and} \quad (c_c + v_c) \frac{n^2}{2}$$

The capital of the combine must be their sum, or:

$$(c_b + v_b + c_c + v_c) \frac{n^2}{2}$$

But this includes the yarn as part of c_c , and that must be deducted.

The..." (D3/12/21/66).

Sraffa ends this document with a hanging "The..." and does not finish the sentence. The last integrated equation above considers the output of the cotton-spinning yarn industry as completely integrated into that of the cotton-weaving cloth industry. In the example considered here both processes are assumed to have the same turnover equal to an annual cycle. What is significant here is a nascent form of reduction where the inputs are conceived as having their own turnover equal to that of the final product. Sraffa juxtaposes the conditions of production (and hence turnover) for each of the commodity processes with that of "society as a whole", once again conceiving that the "average" in many respects represents more than merely the sum of its parts. The question of the "average" of the commodities presents the problem of the "average period of production" and the associated issues regarding its measurement. In the above example since both processes (spinning and weaving) are assumed to be of equal (annual) rotation, the question of averaging is relatively straightforward, even when there is a positive rate of profits. The complications, Sraffa sees, arise in the case of a vertically-integrated combine model where the rotation periods for the respective commodities are no longer equal and uniform. Here Sraffa will begin to address the issues of "balance" in the reduction formula as regards the construction of the "average period" necessary for the solution for "society as a whole". We read from a Nota Bene immediately after the spinning-weaving combine model written 15 September 1942:

15/9/42

N.B.

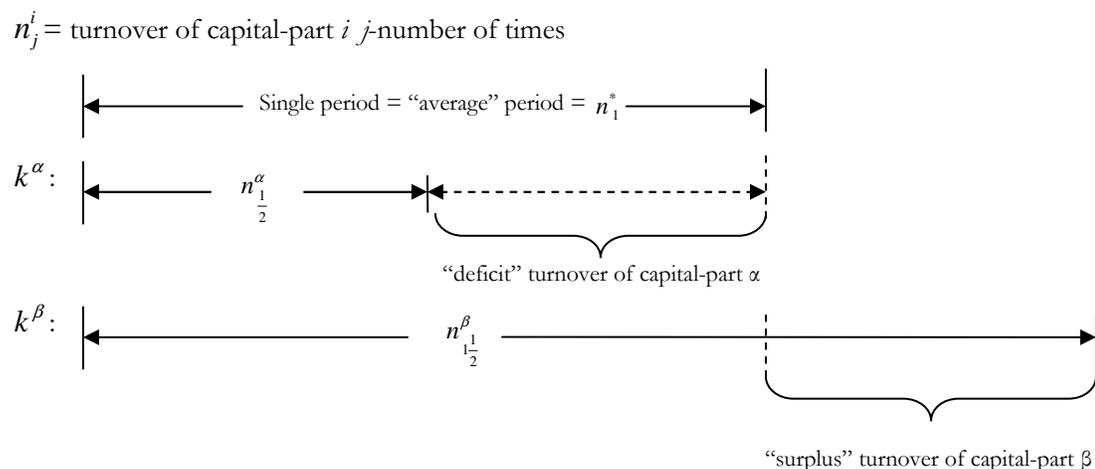
The period of turnover measures a purely material process and cannot possibly be affected by changes in \underline{r} and \underline{w} . That is clear for circ. cap., where each unit of input can be traced to a definite unit of output, whatever \underline{r} may be. And it is equally clear for fixed cap., since the duration of machinery is constant, and so is the annual depreciation independent of \underline{r} and \underline{w} . The confusion begins when we try to average. That is to say, seeing that different parts of a firm's inputs capital have different periods (so that some parts are turned over only once), we try to find a single period during which the whole input capital is turned over only once: the parts that turn over more than once exactly balancing (or compensating) those which turn over less than once in that single period.

Now this balancing of various parts of the input requires a valuation. And this valuation cannot be made before the period has been determined, as it involves the amount of \underline{r} in the particular product. This problem should be solved for society as a whole, on the line of the previous solution. If we do this we shall postulate that the various items composing C have periods of turnover which are independent of whether the commodities comprising them rise or fall with a rise of \underline{r} .

Thus we shall assume $\underline{r} = 0$. Thus we shall find "arbitrary values for p's and an "arbitrary" period of turnover for each firm. such that its whole capital (uno sull' altro *{Italian for one over the other}*) is turned over once in the period, when $\underline{r} = 0$. We can set up the equations, giving the \underline{p} 's and the \underline{n} 's the arbitrary values and find the true \underline{r} for society as a whole." (D3/12/21/68-69).

We can illustrate this balancing of the periods of rotation for a two commodity circulating capital combine model whose "capital-parts" turnover differently, with "capital-part α " turning over in less than one period and "capital-part β " turning over in more than one period. This is seen in Figure 3:

Figure 3: "Balancing" the different turnovers for the different "parts" of a two-commodity combine model (D3/12/21/68-69)



In visualizing the "balancing" of the turnovers for an integrated combine model of production we have taken the liberty to use Sraffa's language of "surplus" and "deficit" he would later employ on his book (and in notes written in the 1950s) such that the construction of the average period results from the off-setting "surpluses" and "deficits". The question of "balancing" in the construction of the "average commodity" will remain for Sraffa up through the construction of the Standard commodity. To see this we fast-forward to a fascinating document that Sraffa wrote on January 28, 1944, *exactly one day after his famous "Hypothesis" document of January*

27, 1944. To set the scene, Sraffa in that aforementioned “Hypothesis” document states unequivocally the exact nature of what his “Hypo” is constructed to accomplish. Here the ratio of constancy is posited in terms of the “repetitive system” such that the “commods. on the left and on the right are in constant ratio” (D3/12/36/61); we can read in this a constant ratio of net product to means of production (maximum, rate of profit) *or* we can also read a constant Casselian quotient. By this time in 1944 Sraffa’s “Average commodity” has undergone a metamorphosis into what Sraffa will now call the “Social revenue”; clearly here this “revenue” will represent the net product of the “Standardized” average commodity. Also by this time Sraffa has realized that he must transform the equations for the actual economic system into a transformed “standardized” system, and here the method by which he accomplished this task is precisely through the reduction formula. We read from the important “Hypotheses” document dated January 27, 1944:

“We solve the original equations for $r = 0$, then for $w = 0$, (*) and obtain R: this R is the measure if the org. comp. we want. If the Soc. Rev. so composed does not satisfy, we take its aggregate Reduction: there must be “humps” or “ditches” in particular years, and these will be due to particular commodities: we eliminate them by multiplying the defective, or dividing the excessive commodity, to whose reduction the “ditch” or “hump” is due.

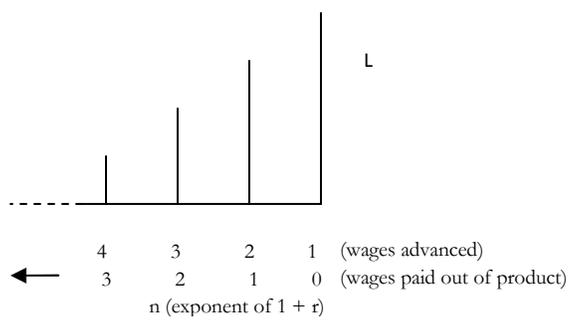
(*) These initial calculations can be made with any commodity as standard: as we have not yet defined standard.” (D3/12/36/66).

We will return to the question of the Organic composition below. However at this juncture what is important is the idea that Sraffa’s use of the reduction methodology allows him to eliminate the “humps” and the “ditches” in particular years; these “humps” and “ditches” are completely analogous to the “surplus” and “deficit” turnovers we illustrate in Figure 3 that arise from the “balancing” of the turnovers in constructing the average period of production in notes written almost a year and a half earlier. Sraffa would draw a very similar graph to our figure 3; instead of “surplus” and “deficit” turnovers horizontally expressed there, Sraffa will vertically-express his “humps” and “ditches”, both for the circulating capital case as well as that of fixed capital, which by the writing of this diagram on January 28, 1944 he had recognized would contain negative quantities.

These notes illustrate quite clearly Sraffa’s project. Here he considers first an actual system of production, which in terms of the reduction would be subject to an anarchy of various turnovers, i.e. humps and ditches. This actual system is subjected to a process whereby the anarchical nature of its composition is “chipped away” in order to render clear the fundamental structural characteristics of that system and in doing so Sraffa “Standardizes” the system. This allows for the unambiguous partitioning of the reduced economic system such that the fundamental constancy can be observed. And observing this fundamental constancy allows for distributional changes to clearly emerge as non-price phenomenon: laws of income distribution can be now be conceived as separate from the theory of value.

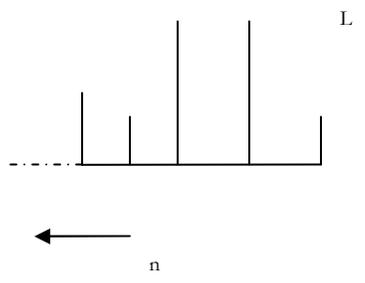
“28.1.44

The Reduction of the System (Standardized) can be represented thus:

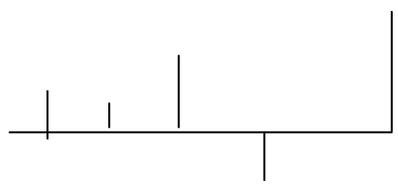


The successive quantities of labour are in constant proportion, such that the ratio of any one of them to the sum of all the preceding ones (i.e. to the left) is R.

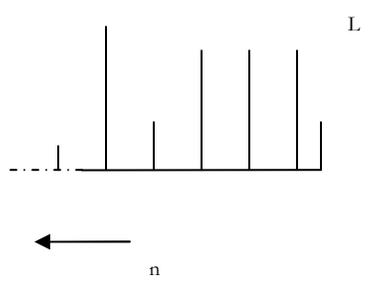
The Reduction of one commod. taken at random may be this:



or (for joint products with negative quantities)



or this



” (D3/12/36/70).

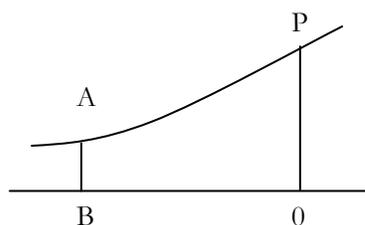
We now turn to the manner in which Sraffa conceives of the organic composition of capital (OCK) in late 1942. We have already seen that he has immersed himself in the reduction methodology; accordingly his conception of the OCK would in this period reflect this. Consider first the manner in which Sraffa adopts the reduction methodology in terms of an interesting diagrammatic depiction using logarithmic curves that relate inputs to output.

“Given that the ratio of the ^[constant] input to output is constant, the real question is: is the value of the capital stock of society as a whole constant of variable with \underline{r} ? is it proportional to input? or is it an intermediate (proportional average?) between input and output?”

Supposing the above answered, how does the capital of any branch of production vary with \underline{r} ? Does it vary simply in proportion to the variations of its own input $(c + v)$? or is there an additional factor of variation?

In general, it is essential to clear up the measure of the “period of turnover”. When $r = 0$ it is simply the value of the stock divided by the ^{value of the} rate of input (which in this case ($r = 0$) is identical with the rate of output).

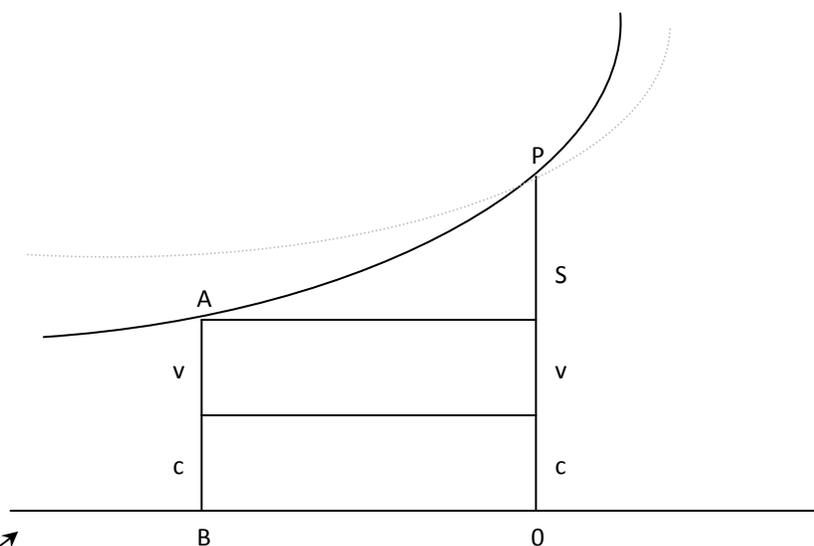
[See logarithmic curve.



The most probable answer for society as a whole is: if $AB = \text{value of rate of input}$ and $PO = \text{rate of output}$, the value of capital stock = area ABOP. When $r = 0$, then $AB = PO$ – Or, is Capital Stock constant with respect to \underline{r} ?” (D3/12/21/70)

This diagram is fascinating indeed. It shows the relationship between input (LHS of graph) and gross output (RHS of graph). The logarithmic relationship shows the transformation of inputs to output at each particular rate of profit. As Sraffa assumes the output to be constant, each logarithmic curve associated with each level of the rate of profit must pass through the output point P. The horizontal axis depicts the particular period of production, or turnover-time. Three pages later we find Sraffa exploring his diagram in more detail:

D3/12/21/73



For society as a whole the ratio of \underline{c} to the output is constant. There is therefore only one source of variation, the ratio of \underline{s} to \underline{v} . But for each branch of production ~~industry~~ the ratio of its particular constant input to its particular output is variable with variations of \underline{r} . There are therefore two [or three? see below] sources of variations, when \underline{r} varies: the amount of wages, and the relative prices of constant input and of output.

[The log. curve AP represents a given \underline{r} . There is a different log. curve (all passing through P, since output is constant for variations of \underline{r}) for each value of \underline{r} .

Now, to eliminate the second source of variation from the individual branch of production, we shall assume temporarily that the ratio of the prices of its particular constant input to that of its particular output is constant for variations of \underline{r} . [In other words, we solve the equations for $r = 0$ and obtain the "arbitrary" values of the p^s .]

[There seems to be still a third source of variation, in the different organic composition of branches of production, the ratio c/v .]

Notice here that Sraffa divides the inputs and output into constant and variable capital and includes the surplus portion on the output side. Different values of the rate of profit are shown by pivoting logarithmic curve of production. We can generalize this idea in the following graph:

Figure 4: Log curve relating inputs and outputs to turnover period (D3/12/21/70, /73, and /76)

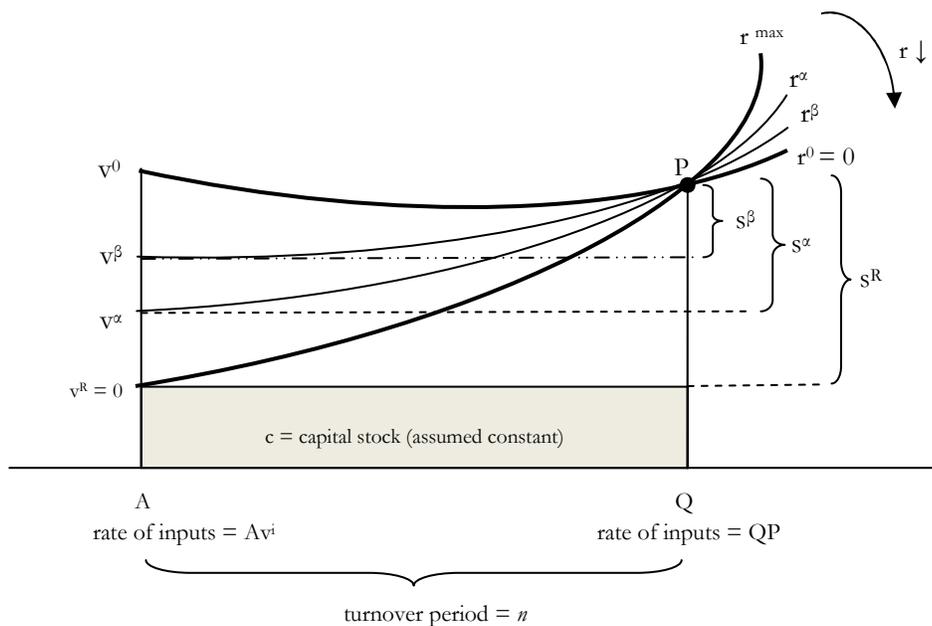
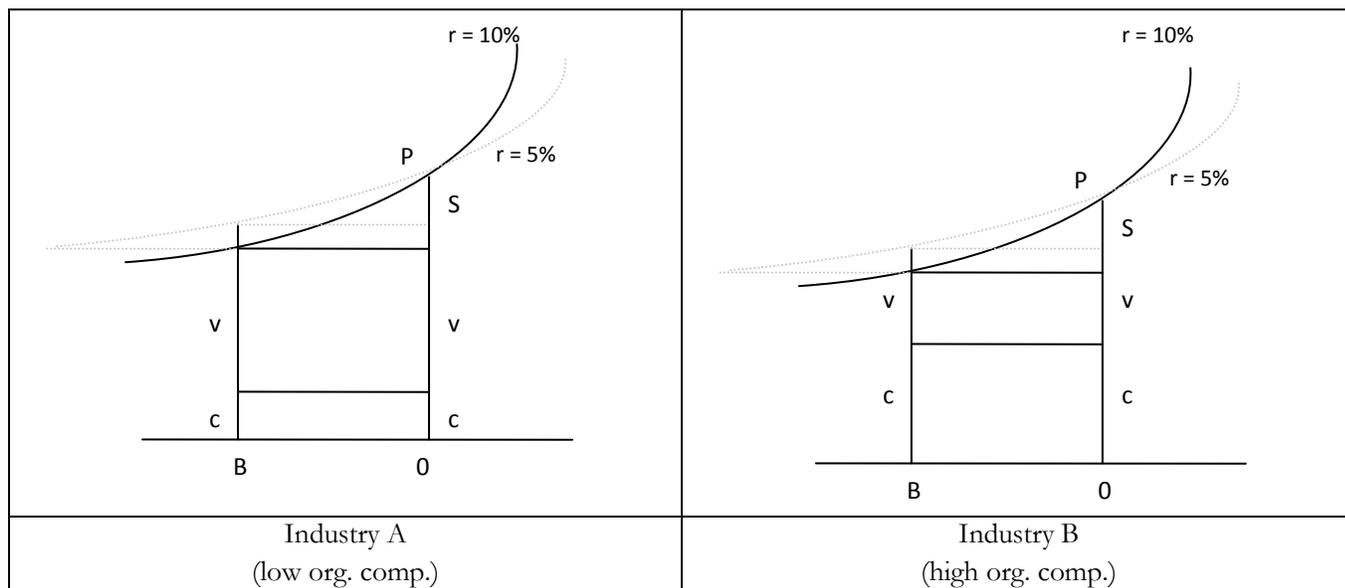


Figure 4 generalizes the proposition that Sraffa is making with his logarithmic diagram. At the maximum rate of profit the “productivity curve” begins where the constant capital stock ends. As the rate of profit moves to its lower limit of zero, the curves pivot about the constant productivity point P. The horizontal axis represents the period of turnover. Sraffa would use this diagram to compare production with different OCK’s but under the assumption of a uniform turnover time:

D3/12/21/76



Such is the state of analysis that Sraffa was steeped at the end of 1942. At the end of the notes on Industrial schemes Sraffa writes a very interesting two-page document that he entitles “Measure of the Organic Composition”.

D3/12/21/1

30.9.42

Measure of Organic Composition

Look at the Industrial Equations, after having found ~~the roots of~~ the arbitrary roots of the p^s and r .

Try to predict how much each commodity will rise or fall with the value of r , without actually finding out the solution; in other words, find the circumstances which determine ^{the extent of} rise or fall.

At first sight there appear 3 sets of circumstances visible in the equations which influence this

- a) Proportion of quantity (value) of commodities to quantity of labour employed in production. Cet. par., the higher this proportion, the more will the commodity rise.
- b) Periods of rotation (the n^s) of the various inputs of an industry. The larger, on the whole, these periods the more the rise.
- c) Composition of the commodities forming input. The more they themselves rise in price with r the more will the product rise.

N.B. Should this start on the Agric. Eq^s, instead of the Industrial ones? It would have the disadvantage of showing at first only two sources of variation. On the other hand it would provide an excellent transition to the Ind. (continuous) model. Or does the “disengagement” process not provide a better transition?

D3/12/21/2

Extra (1)

Suppose, from the industrial equations, we find all the values of r corresponding to all values of w under two methods:

- a) by neglecting all the various values of n ,
- b) in the natural way, taking due account of the proper values of the n^s

We get two functions of w , which we call r_a, r_b .

Query: are the values of r_a and r_b corresponding to any value of w proportional? If not, how are they related?

If they are proportional, the ratio^x is the period of rotation of social capital as a whole. For

$$e^{r_a X} = e^{r_b X}$$

Here we find a more developed idea of the three sources of variation in prices given changes in distribution: (i) the labor contained; (ii) the rotation period; and (iii) the organic composition of capital. He begins to conceive of the “total social capital” as that which marks the division between fluctuations in prices, and formalizes it in terms of his logarithmic diagram, as seen in his equality of the natural exponent e for “social capital” and the individual commodities. The “X” in the exponent represents the proportionality between the

two. In the ensuing fifteen months he would continue to grapple with all of these sources of change, and especially that of the OCK, the latter of which we turn to in the next section.

III. Sraffa and the “corrected” OCK

We have seen in the previous section that in late 1942 Sraffa revisits his equation systems and moves from the case of discontinuous Agricultural Schemes into the case of continuous Industrial Schemes. Key here is the question of the reduction to dated labor that Sraffa embraces with vigor. In conceiving of the Reduction Sraffa begins by conceptualizing the model of complete vertical combines such that he replaces the input-equation of the final goods sector with the equation for the production of those means of production. We see this both in terms of his spinning-weaving combine model found in D3/12/21/66 and also extends the combine model to include fixed capital in his the equations for the production of “ships” found in a 12-page document found in D3/12/26/1-14.

Sraffa conceives of the combine model in notes penned immediately after exposition of the discontinuous and continuous production models in a folder he calls “Notes on ‘Closed Vertical Combines’”, archived as D3/12/22. In these notes, which represent a melding, so to speak, of the Agricultural and Industrial Schemes, Sraffa also begins to conceive of a hypothetical “All-Commodity” equation, which he takes as the sum of all the individual equations per industry. The importance for the “closed combine” model lies in the fact that since all the means of production necessary to produce the final commodity are included as their own equation, these means of production are in effect “reduced” in that the final goods sector merely passes the value of the means of production into the final product and this final product industry in effect “internalizes” the charge for the means of production. We read from the beginning document in this folder:

“Industrial Scheme
(Complete Vertical Combines)

Suppose that each branch of industry consists of a number of complete vertical combines, which produce a commodity of human consumption and produce within themselves all the raw materials, fuel, tools, machinery, transport, etc., required for producing that commodity; so that the only commodity which they buy in the market is the labour force.

The equations on the left will consist exclusively of labour items, each multiplied by $(1 + r)$ at the appropriate power. Although “constant capital” would be absent there would be no difficulty in solving them for each value of \underline{w} . For as soon as a value of \underline{w} is given, the equations become linear, since there are no products of r by the p^s (the former being always on the left, the latter all on the right).

But the extraordinary result is that there is no longer any limit to the rise of \underline{r} . As \underline{w} tends to zero, \underline{r} tends to infinity.

What is so astonishing is that the technical conditions of production may be identical with those of an ordinary system where r has a limit: so that the only difference between the two systems is the way in which the property of various capitals is grouped in different hands. And yet the mere difference in ownership seems to give such an important result” (D3/12/22/1 : 1-2)

We find the last passage very interesting. Notice here how Sraffa distinguishes between the *social* characteristics of the model such that “the mere difference in ownership” of the means of production produces the drastic result that the rate of profit now tends towards infinity. In the “closed vertical combine model” Sraffa revisits the integrated nature of the equation structure along the lines of that developed in his spinning-weaving combine. He develops his “total commodity” equation, conceived as the sum of the equations in the combine, and then proceeds to inquire about the method of Reduction. In D3/12/22/3 : 1 – 4 Sraffa conceives of three separate methods that would carry out the Reduction. He conceives of an annual period of production broken down into 52-week increments. The equations that he uses for this are as follows:

“We obtain a Total equation of the form:

$$(p_a A + p_b B + \dots + p_n N + wL)(1+r)^n + \dots (\dots)(1+r) + \dots + (1+r)^{\frac{1}{52}} = C_t$$

If the distribution among the periods is independent of the composition of the commodities, we can replace the quantities of commodities, of each period, by a quantity of C, equivalent at values for r = 0:

$$(C_n + wL_n)(1+r)^n + (C_{n-1} + wL_{n-1})(1+r)^{n-1} + \dots (C_1 + wL_1)(1+r) + \dots + \left(C_{\frac{1}{m}} + wL_{\frac{1}{m}} \right) (1+r)^{\frac{1}{52}} = C_t$$

(We assume that the smallest period considered is the week, $\frac{1}{52}$)^(x)

^(x) This is justified, as far as L is concerned, by wages being paid at the end of the week: the variable capital beginning to receive profits only from that moment – never therefore for fractions of a week – except where the products is sold in the middle of a subsequent week.” (D3/12/22/1 : 1).

The three distinct methods to effect the Reduction that Sraffa conceives are as follows:

1. “To reduce successively, beginning with those of a shortest period, then going on to the next period, and so on...this process can be regarded as on of ‘sharing back’ the commodity...we should be left with one commodity factor, and a long series of labour factors, from each week of $\frac{1}{52}$ to $(n^2 + n)$ years.
2. To reduce all the items of all periods at the same time: then repeat the operation on all the resulting items, and so on.
3. The third method would be – first to ‘reduce forward’ all the periods longer than the shortest, so as to have a large number of new commodities, each of which is produced by contant and variable capital of the single period of $(1+r)^{\frac{1}{52}}$ ” (D3/12/22/3 : 2-4).

This last method of “reducing forward” has resonance with the idea first proposed by Martin Hill (1933) and discussed by Nurkse (see footnote 19 above). In a set of notes written in November of 1942 (archived as D3/12/26), Sraffa will call this “reduction-forward” by the name of “compounding”. What we find here in the vertical combine model is a juxtaposition of the individual industries alongside that of the All-Commodity. This idea Sraffa will return to in the next set of notes he entitles simply “Reduction” (archived as D3/12/24).

The folder “Reduction” contains three separate independent documents. The chronologically first is a 21 page set of notes that he gives various titles to; from “General equation for ‘Average Commodity C’, to ‘Reduction of commodity B”, to “constant of each commodity made of average commodity” itself”, and finally “Constant made of itself”. The latter two conceptions are very fascinating indeed, and will in our assessment provide Sraffa with the basis for his eventual “correction” of Marx’s OCK. The idea that Sraffa arrives at in these notes is the conception of a commodity completely date-reduced whose means of production requirements are composed completely of “Average commodity” (D3/12/24/15). This is drafted on October 3, 1942, and we find here how Sraffa starts to conceive of his Hypothesis of constancy. He will then three days later on October 6, 1942 move from a commodity whose means of production requirements are composed of “Average commodity” to the Reduction of the population of commodities whose constant capital of each “consists entirely of itself” (D3/12/24/20). We begin to discern here the homogeneity relationship at the level of the individual commodities that would come to characterize his Standard system. The actual Reduction equations that Sraffa would arrive at are found on the 21st page of this document that he entitles “Reduction for any Commodity B”:

“Reduction for any Commodity B
(cont. from p. 2)

(Try to apply to this case, as far as possible, forms of pp. 3-5)

The original equation is (w and p^s) in terms of Average Comm)

$$(p_a A_b + p_b B_b + \dots + p_n N_b)(1+r) + wL_b(1+r) = p_b B_b$$

Call the series in first bracket $\sum_1 A, \dots N$

Replace each of the commodities in the first bracket by its components (i.e. the left h.s. of its own

equation ~~divided by~~ multiplied by respectively divided by the ratios $\frac{A_t}{A_b}, \frac{B_t}{B_b}, \text{etc.}$).

Collect all the commodities in one bracket all the quantities of labour in another and, multiplied by its proper profit factor replace in this equation

$$\left\{ \frac{A_b}{A_t} (p_a A_a + p_b B_a + \dots + p_n N_a) + \frac{B_b}{B_t} (p_a A_b + \dots + p_n N_b) + \frac{N_b}{N_t} (p_a A_n + \dots + p_n N_n) \right\} (1+r)^2 + \left(\frac{A_b}{A_t} wL_a + \frac{B_b}{B_t} wL_b + \dots + \frac{N_b}{N_t} wL_n \right) (1+r)^2 + wL_b = p_b B_b$$

Call the series in the first bracket $\sum_2 A, \dots N$ and the labour series $\sum_2 L$. We can then write

$$\left(\sum_2 A, \dots N \right) (1+r)^2 + \left(\sum_2 wL \right) (1+r)^2 + wL_b (1+r) = p_b B_b$$

Repeat the substitution of the items in the commodity series n times, we have

$$\left(\sum_n A, \dots N\right)(1+r)^n + \left(\sum_n wL\right)(1+r)^n + \left(\sum_{n-1} wL\right)(1+r)^{n-1} + \left(\sum_{n-2} wL\right)(1+r)^{n-2} + \dots + \left(\sum_2 wL\right)(1+r)^2 wL_b(1+r) = p_b B_t$$

The number of items in the commodity bracket grows at every substitution: if m is the average number entering each of the commodities in question, the number of items after n substitutions will be m^n . But their total value decreases, being replaced in our equation by the increasing total value (as well as number) of labour items. When the process of substitution has been carried far enough [define]...we shall have very large number of extremely small commodity items; and they will include all (?) commodities used in production in proportions quite different from those in which they entered B directly. This mass of commodities we can regard as a random sample of all commodities, and we can assume that the price ~~value~~ of the aggregate in relation to the average commodity will not vary with r . We can therefore replace this aggregate with a quantity of equal value of the Average commodity (taking all values at ~~for~~ $r = 0$):

$$C_s(1+r)^n + \left(\sum_n wL\right)(1+r)^n + \left(\sum_{n-1} wL\right)(1+r)^{n-1} + \dots + \left(\sum_2 wL\right)(1+r)^2 + wL_b(1+r) = p_b B_t$$

[Change of notation (to be ins. from p. 9): replace $\sum_n wL$ with $wL_n, etc.$]

$$C_s(1+r)^n + wL_n(1+r)^n + wL_{n-1}(1+r)^{n-1} + \dots + wL_2(1+r)^2 + wL_b(1+r) = p_b B_t$$

Replace all the w's with the expression found ^{from the general equations} on p. 8 bris $w = \frac{C_t - C_c(1+r)}{L_c(1+r)}$

$$C_s(1+r)^n + \frac{L_n}{L_c} \{C_t(1+r)^{n-1} - C_c(1+r)^n\} + \frac{L_{n-1}}{L_c} \{C_t(1+r)^{n-2} - C_c(1+r)^{n-1}\} + \dots$$

$$+ \frac{L_2}{L_c} \{C_t(1+r) - C_c(1+r)^2\} \frac{L_b}{L_c} \{C_t - C_c(1+r)\} = p_b B_t$$

Since (p. 3) $C_c = \frac{C_t}{1+r_{\max}}$ all these terms, except the first (the original "commodity" term) become =0 when $r = \text{maximum}$, i.e. when $w = 0$."

In these notes we find Sraffa date-reducing "any commodity B" such that this reduction will be effectuated for *all* commodities in the economic system. And this proves to be the essence of the "correction" that he would make in the OCK beginning in February 1944.

And this is precisely the point. On January 27, 1944 Sraffa would write the famous "Hypothesis" document that has been cited in the previous literature (Gilibert 2003; de Vivo 2003; Bellofiore 2008, 2010; Kurz 2006; Gehrke and Kurz 2006; Kurz and Salvadori 2008, 2010). The "Hypothesis" document itself consists of 21 continuous pages in which Sraffa restates after 15+ months of intense study the original Hypothesis he had been developing since as far back as late 1942. In this document Sraffa speaks to how Marx himself "knew

all of this” with regard to the question of the aggregate Social Revenue and the idea of the average OCK. He develops a “Standardized Social Revenue”, which is but the nascent form of the Standard Net Product. He moves this Social Revenue through backward-dating via the Reduction and “regularizes” each successive back-date into smaller and smaller date-reduced labor components. He recognizes that the “commodity residue” will always remain, and premises the whole question of the Maximum rate of profit to this “all-important” residue component.

After going through the details of his “Hypothesis” revisited after years of intense study, on February 5, 1944 he makes the startling realization that the construction of the OCK that is in Marx was in need of a fundamental “correction.” The basic correction for Sraffa becomes one of *not* conceptualizing the OCK as in Marx where the constant capital component represents the value of the heterogeneous means of production requirements per industry. Instead, Sraffa now conceives of a “corrected” measure that would take as the constant capital component the quantum of means of production of a particular commodity (“any commodity B”) *used in the economy as a whole*. We find here the methodology he would employ in constructing his Standard system. The relevant document is archived as D3/12/36/60 : 1-7, and thus is a seven page document that develops this “new” OCK measure. We read from the beginning of that document:

1

“5.2.44

Organic Composition of Capital

Corrected Definition

So far I have defined the Org. Comp. of the capital producing a commodity A, as the ratio of the value of the commodities used up in its production to the quantity of labour employed in producing it. E.g. if the equation of that commodity is

$$(A_a p_a + \dots + K_a p_k + Lw)(1 + r) = A p_a$$

The Org. Comp. was the ratio (for $r = 0$) of $\frac{A_a p_a + \dots + K_a p_k}{Lw}$.

This made the Org. Comp. comparatively unusable, for so much depended on the variations in price of those commodities.

The correct definition is:

The Org. Comp. of the capital producing a commodity A is the (old definition) Org. Comp. of the Social (aggregate) Capital of a ^{repetitive} system producing with the given methods of Social Revenue entirely composed of commodity A.

In other terms it is the capital necessary to produce a given quantity of A and to reproduce all its means of production.”

For Marx, the OCK was defined as the price (value) of the means of production of the industry (constant capital) to the price (value) of the labor purchased (variable capital). In a multi-commodity (basic) world, the means of production requirements will of necessity include elements of all other basic goods. To commensurate, a principle of valuation is required. We are, in other words, stopped by the word “price”. In a two sector model, Marx’s OCK thus can be defined as:

$$OCK_i^{KM} = \frac{[p_i A_{ii} + p_j A_{ji}]}{\omega L_i}$$

$$OCK_j^{KM} = \frac{[p_i A_{ij} + p_j A_{jj}]}{\omega L_j}$$

Sraffa begins to consider the question in a different way. He begins to wonder if there is a way to render uniformity across all commodity subsystems. This arose through the idea of the reduction for “any commodity B”. Thus we can reduce *all commodity B*’s to have the same ratio of the price (value) of the commodity-as-capital used as means of production system-wide to the price (value) of labor purchases (variable capital). Sraffa’s OCK can thus be depicted as:

$$OCK_i^{PS} = \frac{[p_i A_{ii} + p_i A_{ij}]}{\omega L_i} = \frac{p_i \sum_{k=i}^j A_{ik}}{\omega L_i} = \frac{p_i A_i}{\omega L_i}$$

$$OCK_j^{PS} = \frac{[p_j A_{ji} + p_j A_{jj}]}{\omega L_j} = \frac{p_j \sum_{k=i}^j A_{jk}}{\omega L_j} = \frac{p_j A_j}{\omega L_j}$$

Notice here that the price in the numerator of each OCK^{PS} is the same – hence here price no longer stops us. And it is *this* ratio that Sraffa equalizes in the construction of his Standard system.²⁵ In no way does this alter the difference in the OCK of Marx; indeed this is precisely Sraffa’s point. We can have equality in Sraffa’s OCK:

$$(OCK_i^{PS} = OCK_j^{PS} = \dots = OCK_n^{PS});$$

and still have inequality in Marx’s:

$$\left(OCK_i^{KM} \begin{matrix} \leq \\ \geq \end{matrix} OCK_j^{KM} \begin{matrix} \leq \\ \geq \end{matrix} \dots \begin{matrix} \leq \\ \geq \end{matrix} OCK_n^{KM} \right).$$

And it is precisely the relationship between the equality of OCK^{PS} and the inequality of OCK^{KM} that Sraffa locates the question of surplus and deficit industries and the attendant transfers of value that are necessary due precisely to unequal OCK^{KM} . By the end of February Sraffa would have constructed his q -system of

²⁵ In actual fact the Standard ratio (ζ) is defined as the ratio of physical net product (Y_{npi}) to the physical system-wide means of production requirements (A_i); note the ratio zeta (ζ) is a physical ratio at the level of the subsystem. When a system is expressed in its Standard proportions, uniformity of the zeta’s ($\zeta_i = \zeta_j = \dots = \zeta_{\text{Standard}}$) holds throughout. Pasinetti (1977) calls this ratio the “commodity own-rates of reproduction”.

quantity multipliers, something that he was able to do because he had earlier understood the equality across commodity sub-systems he was looking for (D3/12/36/5). From this point on Sraffa became increasingly reticent with respect to the date-reduction framework, although he continued to struggle with it even up through the Majorca Draft of March 1955 (D3/12/52). As we know he ultimately chose in his book to adopt the q -system physicalist framework first (Chapter I – V) and only later the date-reduced framework (Chapter VI; it is of interest to note that in this single sixth chapter Sraffa reproduces the logic of all the propositions contained in the first five).

IV. Preliminary Conclusion

What we therefore discover is that Sraffa constructs of the concept of the organic composition of capital (OCK) in a manner different from that of Marx. This Sraffa accomplishes in a series of notes written in February 1944, and calls his measure of the OCK the “corrected” one; hence Sraffa “corrects” Marx. The basic idea behind the notion of the OCK that Sraffa develops conceptualizes the ratio at the *intra-commodity* level, whereas Marx’s OCK conceptualizes the ratio at the *inter-industry* level. In terms of the two sector model the distinction can be written as follows:

$$OCK_i^{KM} = \frac{[A_{ii}p_i + A_{ji}p_j]}{\omega L_i} \neq \frac{[A_{ii}p_i + A_{ij}p_i]}{\omega L_i} = OCK_i^{PS}$$

$$OCK_j^{KM} = \frac{[A_{ij}p_i + A_{jj}p_j]}{\omega L_i} \neq \frac{[A_{ji}p_i + A_{jj}p_i]}{\omega L_i} = OCK_i^{PS}$$

The difference of course is in the numerator. For Marx, the numerator consists of the value of the heterogeneous commodity inputs that are used in an industry. For Sraffa the numerator consists of the homogenous quantities of means of production necessary for systemic production. Marx’s measure necessarily requires a principle of valuation in order to commensurate the heterogeneous means of production inputs; in a word, Marx here is “stopped by the word price”. The same does not apply to Sraffa’s measure. There the numerator consists of the same commodity – it is *intra-commodity* in nature. Hence price does not stop us. And it was along these lines that Sraffa maintained that the rate of profit was a non-price phenomenon, and further that this “physicalist” notion of the distribution of the surplus provided for a conceptual “rock to cling to” when the system of equations for the actual system faces the anarchy of price changes with changes in income distribution.

The manner in which Sraffa accomplished this is very interesting. We saw that he begins first with the notion of date-reduced labor of the Austrian period of production approach, especially that of Böhm-Bawerk and Wicksell. In notes entitled “Reduction for any commodity B” written around September 1942 Sraffa pens a series of reduction equations. One interesting version appears as (D3/12/24/27):

“Reduction for any commodity B”

$$\underbrace{\left(\sum_n A, \dots N\right)(1+r)^n}_{\text{commodity residue}} + \underbrace{\left(\sum_n wL\right)(1+r)^n + \left(\sum_{n-1} wL\right)(1+r)^{n-1} + \left(\sum_{n-2} wL\right)(1+r)^{n-2} + \dots + \left(\sum_2 wL\right)(1+r)^2}_{\text{date-reduced means of production requirement}} + \underbrace{wL_b(1+r)}_{\text{direct labor}} = p_b B_t$$

The commodity is conceptualized as the sum of direct labor (here with wages paid ante-factum)²⁶, the date-reduced means of production requirements, and a commodity residue that would tend towards zero but never actually get there. This latter point became the basis of the maximum rate of profit concept that Sraffa got from Marx. What he will show in the ensuing years is that the maximum rate of profit is precisely the social OCK.

We can construct the date-reduced OCK for any commodity B measure by using Sraffa’s reduction equation.

$$OCK_b^{PS} = \frac{wL_b(1+r)^2 + \left(\sum_n wL\right)(1+r)^n + \left(\sum_{n-1} wL\right)(1+r)^{n-1} + \left(\sum_{n-2} wL\right)(1+r)^{n-2} + \dots + \left(\sum_2 wL\right)(1+r)^2 + \left(\sum_n A, \dots N\right)(1+r)^n}{wL_b(1+r)}$$

Rearranging and cancelling out the profit factor yields:

$$OCK_b^{PS} = \frac{wL_b(1+r) + \left(\sum_2 wL\right)(1+r)^2 + \left(\sum_{n-2} wL\right)(1+r)^{n-3} + \left(\sum_{n-1} wL\right)(1+r)^{n-2} + \left(\sum_n wL\right)(1+r)^{n-1} + \dots + \left(\sum_n A, \dots N\right)(1+r)^{n-1}}{wL_b}$$

$$OCK_b^{PS} = \underbrace{\frac{wL_b(1+r) + \left(\sum_2 wL\right)(1+r)^2 + \left(\sum_{n-2} wL\right)(1+r)^{n-3} + \left(\sum_{n-1} wL\right)(1+r)^{n-2} + \left(\sum_n wL\right)(1+r)^{n-1}}{wL_b}}_{\text{Date-reduced OCK in terms of “any commodity B”}} + \dots + \underbrace{\frac{\left(\sum_n A, \dots N\right)(1+r)^{n-1}}{wL_b}}_{\text{Commodity residual}}$$

This then becomes the basis for Sraffa’s “corrected” OCK. The key behind this is the idea that the correct OCK conceptualizes an economic system by arranging the means of production requirements at the intra-commodity level. Sraffa conceived of this OCK measure as the ratio of the embodied (here date-reduced) labor of the means of production – what he terms Ricardo’s labor embodied measure – to the living labor added – what he terms Mat6hus’s measure. At the end of this seven page document we find this explicit:

²⁶ Kurz (2006, pg. 16) identifies late 1943 as the time when Sraffa had chosen to treat the wage as a share paid out of the product – i.e. post factum.

D3/12/36/60 (6)

“16.2.44

Org. Comp.

From prec. p. this follows.

The Org. Comp. [when properly defined, as above] and the average Period are the same thing [i.e. are proportional to one another].

This can be shown either from the Reduction or from “the Sub-System whose net product consists entirely of the commodity in question.” [These two are interchangeable: the Sub-System is obtained by “de-reducing” the Reduction series of the commodity in question]

Neither can be obtained solely by considering the equation of the particular commodity – even if the Values of the means of production are known. Both require bringing in the whole of the basic equations, i.e. the Standard System.

- I. Take Reduction. The sum of the quantities of labour (the L's) gives the value of the commodity: this is equal to the living labour of the Sub-System, i.e. to the variable capital (in Malthus's measure, labour commanded)

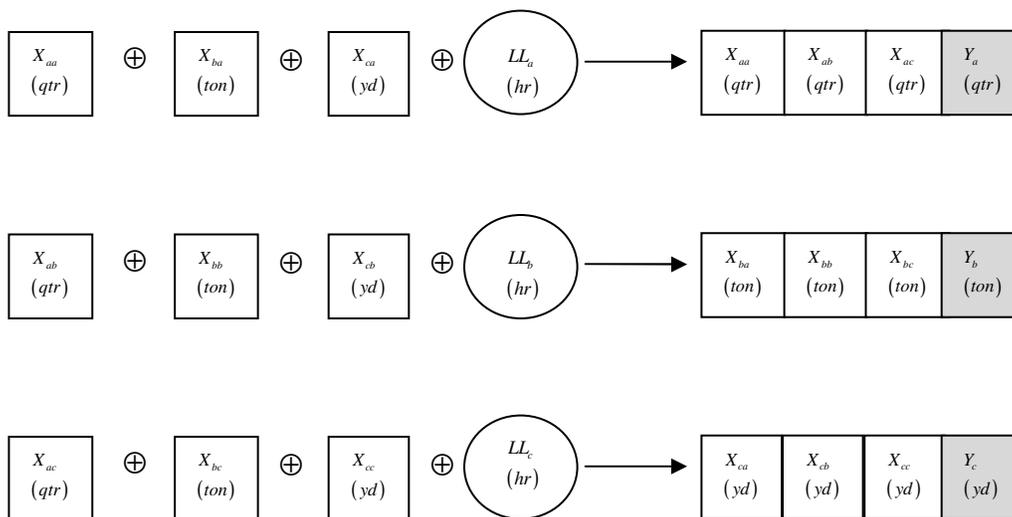
The sum of “the quantities of labour (the L's) each multiplied by the exponent of its respective profit factor $(1 + r)$ ” gives the Values of the constant capital (in Ricardo's measure, labour embodied).

The ratio of the two is the Organic Composition of capital which produces the commodity.”

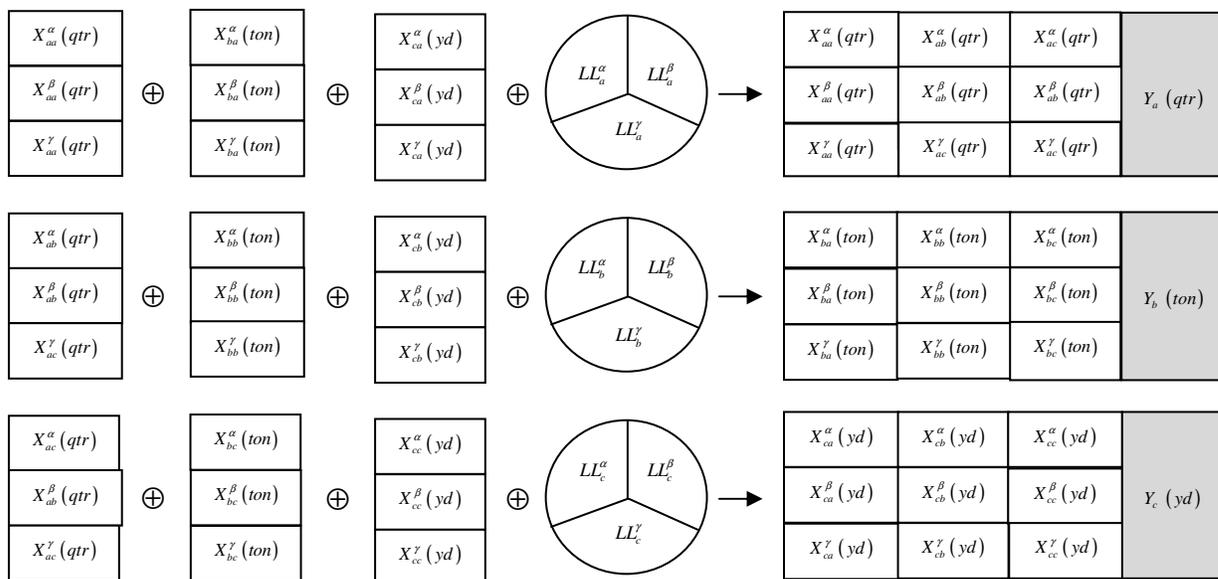
Therefore Sraffa's OCK concept can be seen as the conceptual unity of the two different measures of labor that appeared in Classical Political Economy, namely the ratio of Ricardo's labor embodied measure to Malthus's labor commanded one.

(Nov. 22, 2010: This work is to be finished...)

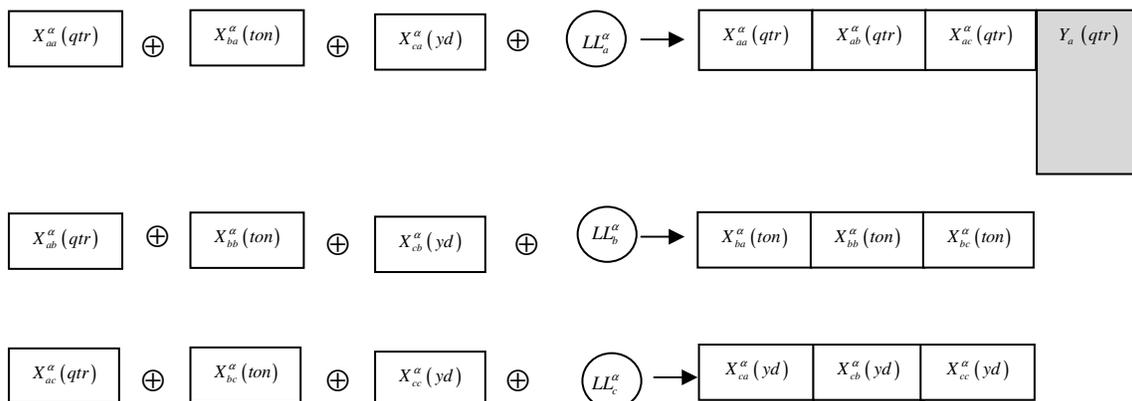
Appendix A: Single Product Industries Economic System (Actual or Standard)
(Adapted from Harcourt and Massaro (1964))



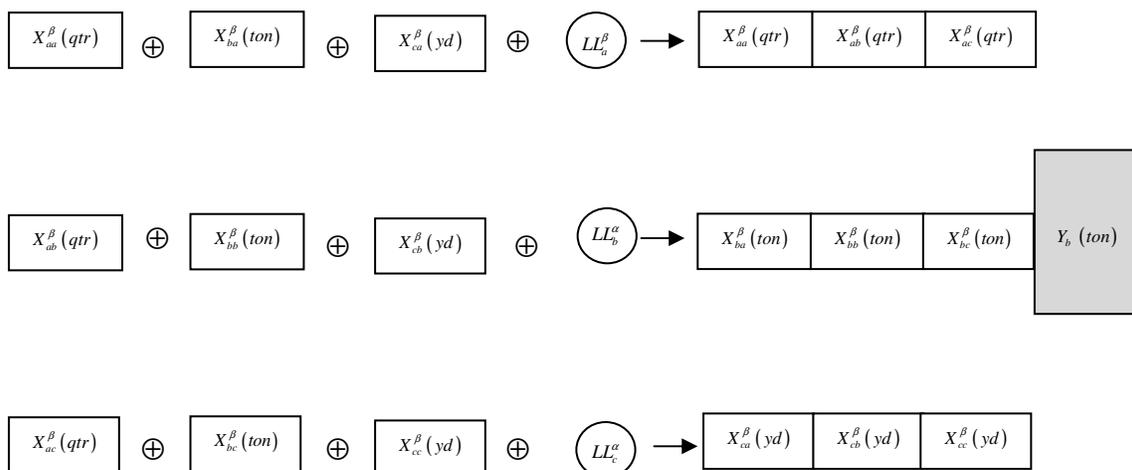
Sub-systems for economic system



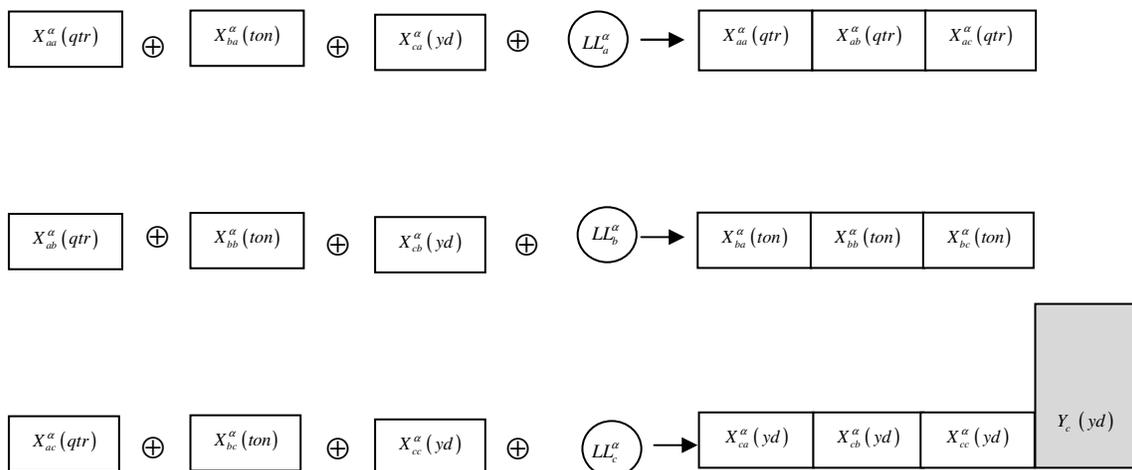
Sub-system α for the production of one unit net output good a:



Sub-system β for the production of one unit net output good b:



Sub-system γ for the production of one unit net output good c:



Appendix B: Concordance for Sraffa Papers D3/12/21: Industrial Schemes

Sraffa's pagination	Archival number	Sraffa's pagination	Archival number
1	D3/12/21/57	32	D3/12/21/46
2	D3/12/21/58	33	D3/12/21/43
3	D3/12/21/59	34	D3/12/21/44
4	D3/12/21/60	35	D3/12/21/41
5	D3/12/21/61	36	D3/12/21/42
6	D3/12/21/62	37	D3/12/21/39
7	D3/12/21/63	38	D3/12/21/40
8	D3/12/21/64	38 bis	D3/12/21/37
9	D3/12/21/65	39	D3/12/21/38
10	D3/12/21/66	40	D3/12/21/36
11	D3/12/21/67	41	D3/12/21/35
12	D3/12/21/68	42	D3/12/21/34
13	D3/12/21/69	42 bis	D3/12/21/33
14	D3/12/21/70	43	D3/12/21/30
15	D3/12/21/71	43 bis	D3/12/21/28
16	D3/12/21/72	43 ter	D3/12/21/27
17	D3/12/21/73	43 ter	D3/12/21/29
18	D3/12/21/74	43 quarto	D3/12/21/26
19	D3/12/21/75	43 quinto	D3/12/21/25
20	D3/12/21/76	44	D3/12/21/23
21	D3/12/21/77	44	D3/12/21/31
22	D3/12/21/78	44bis	D3/12/21/24
23	D3/12/21/53	44 ter	D3/12/21/21
24	D3/12/21/54	45	D3/12/21/20
25	D3/12/21/51	45 bis	D3/12/21/19
26	D3/12/21/52	45 ter	D3/12/21/18
27	D3/12/21/49	45 quattro	D3/12/21/11
28	D3/12/21/50	46	D3/12/21/12
29	D3/12/21/47	47	D3/12/21/10
30	D3/12/21/48	47 etc	D3/12/21/3
31	D3/12/21/45	Sraffa's Original Folder	D3/12/21/81

Works Cited

- Bellofiore, Riccardo (2008). Sraffa after Marx: An open issue, in Chiodi, G. and Ditta, L. *Sraffa or an Alternative Economics*, Palgrave Macmillan.
- Bellofiore, Riccardo (2010). The ‘tiresome objector’ and Old Moor: A renewal of the debate of Marx and Sraffa based on the unpublished material at the Wren Library, Paper presented at Cambridge Journal of Economics Celebratory Session *New Perspectives on the work of Piero Sraffa*.
- Carter, Scott (2009). From “pool of profits” to surplus and deficit industries: Archival evidence on the evolution of Piero Sraffa’s thought. Unpublished MS, The University of Tulsa.
- Carter, Scott (2010). Sraffa on income distribution in a capitalist system, Paper presented at Cambridge Journal of Economics Celebratory Session *New Perspectives on the work of Piero Sraffa*.
- Cassel, Gustav 1935. *On quantitative thinking in economics*, Oxford: Clarendon Press.
- De Vivo, G. 2003. Sraffa’s path to *Production of Commodities by Means of Commodities*: an interpretation, *Contributions to Political Economy*, vol. 6, pp. 1-25.
- Fleming, J. Marcus (1933). The period of production and derived concepts, , *Review of Economic Studies*, Vol. 3, no. 1, pp. 1-17.
- Gehrke, C. and Kurz, K. 2006. Sraffa and Bortkiewicz, *History of Political Economy*, vol. 38, no. 1, 91
- Gifford, C.H.P. (1933). The concept of the length of the period of production, *Economic Journal*, Vol. 43, no. 172, pp. 611-618.
- Gilibert, G. 2003. The equations unveiled: Sraffa’s price equations in the making, *Contributions to Political Economy*, vol. 22, pp. 27-40
- Harcourt, G.C. and Massaro, Vincent (1964). A note of Mr. Sraffa’s subsystems, *Economic Journal*, Vol. 74, pp. 715-722.
- Hill, Martin (1933). The period of production and industrial fluctuations, *Economic Journal*, Vol. 43, no. 172, pp. 599-610.
- Howard, M. C. (1980). Austrian capital theory: An evaluation in terms of Piero Sraffa’s *Production of commodities BY Means of Commodities*, *Metroeconomica*, Vol. 32, no. 1, pp. 1-29.
- Kurz, H. D. and Salvadori, N. 2010. Sraffa and the labour theory of value, pp. 187-213 in Vint, J., Metcalfe, J. S., Kurz, H, Salvadori, N and Samuelson, S. (eds.) *Economic theory and economic thought: essays in honour of Ian Steedman*, London, Routledge
- Kurz, Heinz and Salvadori, Neri (2008). On the collaboration between Sraffa and Besicovitch: The ‘proof of gradient’, in Chiodi, G. and Ditta, L. *Sraffa or an Alternative Economics*, Palgrave Macmillan.
- Kurz, Heinz. 2006. The agents of production are themselves commodities: on the classical theory of production, distribution, and value. *Structural Change and Economic Dynamics*, vol. 17, pp. 1-26

- Lager, Christian. (2000). Production, prices, and time: A comparison of some alternative concepts, *Economic Systems Research*, Vol. 12, no. 2, pp. 231-253.
- Marcuzzo, Maria Cristina (2005). Piero Sraffa at the University of Cambridge, *European Journal of the History of Economic Thought*, Vol. 12, no. 3.
- Marschak, J. A note on the period of production, *Economic Journal*, Vol. 44, no. 173, pp. 146-151.
- Mitchell, Wesely, (1969). *Lecture notes on types of economic theory*.
- Naldi, Nerio (2005). Piero Sraffa: Emigration and scientific activity, 1921-1945. *European Journal of the History of Economic Thought*, Vol. 12, no. 3, pp. 379-402.
- Nurkse, Rangar (1935). The Schematic representation of the structure of production, *Review of Economic Studies*, Vol. 2, no. 3, pp. 232-244.
- Pasinetti, Luigi. (1977). *Lectures on the theory of production*, Columbia University Press.
- Smith, Jonathan (1998). An Archivist's apology: the papers of Piero Sraffa at Trinity College, Cambridge, *Il Pensiero Italiano*, VI, 1, pp. 39-54.
- Sraffa and Dobb 1951. *The Works and Correspondence of David Ricardo*, Eleven Volume, Cambridge University Press: Cambridge
- Sraffa, P. (1960). *Production of Commodities by Means of Commodities*, CUP.
- Wicksell, Knut. (1934) *Lectures on Political Economy* Two Volumes. George Routledge and Sons.