

Sraffa on income distribution in a capitalist system¹

1. Introduction

Sraffa's (1960) approach to the question of income distribution has long been a source of varied interpretation. What is recognized by all parties is the fact that in *Production of Commodities by Means of Commodities* the distribution parameter remains open; this is to say it may be closed by either the wage-side or the profit-side of the ledger. Sraffa does advance closure in his system by the profit relation with his cryptic reference to the "money rate of interest" in his famous paragraph 44. But this is by no means a definitive closure for Sraffa, meaning that closing the system from the perspective of financial markets, etc. is *not* for Sraffa unequivocal. And that's the beauty of his analytical price-theoretic framework. Because the distribution parameter remains open, it immediately gains a robust quality – i.e. Sraffa develops a more *general* framework. Indeed plausible and perhaps not incompatible theories of distribution that consider *both* wage-type *as well as* profit-type closure mechanisms can be articulated and advanced from this more general and robust framework. And here we are in agreement with Pasinetti (1988, p. 135), who in the *CJE*'s Memorial Issue argues that Sraffa "is concerned, in an essential way, with the *foundations* of a theory of income distribution". Thus Sraffa does develop the theory of income distribution, specifically the *foundations* for a variety of approaches steeped in the "submerged and forgotten" Classical theory. But since Sraffa's contributions are essentially foundational, no specific theory broadly within the surplus approach to value and distribution can be singled-out, we contend, as "the" theory of distribution that Sraffa advanced.

This essay attempts to utilize Sraffa's foundations for an interpretation within the Marxian tradition, specifically related to the exploitation theory of profits. [It is acknowledged from the outset that the approach advanced here is *our* Marxian reading of Sraffa's foundations, and not necessarily that of](#)

Sraffa himself. In this reading, income distribution is defined as the division of the net product, or value-added by living labour, as wage vs. profit revenues. From the Marxian perspective, this requires the logical distinction between *extraction* of profits from workers to that of *distribution* of these profits to the owners of capital advanced. Clearly this requires a theory of exploitation. And recent evidence from the Sraffa Papers at Trinity College, Cambridge, demonstrates that Sraffa indeed did explicitly operationalize a theory of exploitation in the development of his theory of price, as seen specifically in the concept of the “pool of profits” he retained over the crucial 14 year period from 1942 to 1956. Study of archival material shows that Marx’s theory of value, exploitation, and distribution greatly influenced Sraffa’s own intellectual development; or perhaps better said, in significant respects Sraffa utilized the analytical framework of Marx to conceptualize his economic system in much of the same way.²

The remainder of this essay is as follows. The first section discusses in brief openness of the distribution parameter in Sraffa’s system. The second section discusses the uniquely Classical methodology that Sraffa adopted in terms of the manner of his inquiry, specifically the importance on an analytical plane to distinguish generalized commodity production from capitalist commodity production. The third section develops some of the implications in the two systems of production and it is shown that even within the no-surplus society, fundamental conditions exist establishing conservation of value from production to exchange. In the present paper we will use the Classical terminology *bestow* (or *embody*) and *command*. Hence new value is *bestowed in production*, and the product of this productive labour (net product) is *commanded in exchange*. In the fourth section the issue of *bestow* and *command* will be extended to the case of capitalist commodity production, wherein the remuneration to the different “factors” of production produces fundamental asymmetries that in turn affect the system of natural prices (prices of production). The asymmetries associated with

factor remuneration express themselves as asymmetries in the (natural) price form, and here we find the concepts of surplus and deficit industries clearly emerge, ultimately Sraffa's own characterization of the transformation from values to prices. The final section concludes.

1. Sraffa and the close of his system

Sraffa laid the foundations for alternative theories of income distribution. The location of these foundations reside, we contend, in the question of how the Sraffa system is closed, whether on the wage side or profit side of the ledger, and what institutional mechanisms may be necessary to effectuate these alternative closures. He left this problem for the next generations of economists to wrangle with. The only two unequivocal results on this score that come out of his political economy, both in terms of his book as well as his archival notes, are (i) the negative conclusion that whatever the true source, closure does not emanate from marginal productivity; and (ii) that it is “susceptible of being determined from outside the system of production” (Sraffa, 1960 p. 33).³

The notion of “outside the system of production” is open to varied interpretations. This “outside closure” could be either wage-type or profit-type – both are *theoretically* possible within the framework developed by Sraffa. The Ricardo-Malthus demographic equilibrium story certainly is outside the production system, as is the related given wage-bundle approach of both “subsistence” as well as “custom.” Here the distribution parameter is closed by the wage relation, with profit type revenues residually determined, and both are compatible within Sraffa's theoretical framework. On the profit-side of the coin, Sraffa remarks that the system can be closed by the money rates of interest. Institutional mechanisms can also exist that allow for “outside” closure. The most notable is that of trade union activity which closes the distribution parameter from the wage side of the coin (see D3/12/36/88 : 1). And although not in his book, Sraffa identifies in drafts of the famous passage on the money rate of interests two financial institutions, the Stock Exchange and the

Banking sector, as effectuating closure on the profit side (see D3/12/68/1-2 and D3/12/78/6).

But what was the key for him, we contend, was not necessarily *how* distribution was closed, but rather that of the consequent effect on price formation given the changes in distribution effectuated by the closure.

So where does the Marxian approach fit in, the subject of this essay? We are of the opinion that the Marxian theory of exploitation is compatible with *either* closure on the wage-side *or* the profit-side of the income distribution parameter. Debates related to this within the Marxian paradigm concern the commodity-bundle approach vs. the share approach to the value of labour power, articulated within the context of Sraffa's framework first by Eatwell (1975, p. 550) and continuing in the contemporary discussions in the debate of the Dumenil-Foley New Interpretation to the transformation (Foley, 2000, Perri 2010). Hence what makes our approach to Sraffa's paradigmatic foundations "Marxian" in character has nothing to do with how the system closes. Rather, important for us is the fact that the system *is* closed in the sense that the share of income received by workers is less than the productivity of their labour. And this for us characterizes the *capitalistic* character of the income distribution story in Sraffa's analytical framework, the main subject of this essay and to which we now turn.

2. Production under different regimes of distribution

Consistent with the manner of inquiry in Classical theory, Sraffa felt it incumbent to develop the value-distribution problematic in capitalistic systems by first conceptualizing an "ideal type" system of production wherein the problem of distribution did not enter – i.e. when the direct producers received the entire gains of their labour.⁴ For Sraffa this meant a situation when the wage share (ω) is unity; or stated precisely, the economic system in Sraffa's "ideal type" operates under a unitary wage share regime of functional distribution. Far from trivial, unitary wage share price systems have

tremendous heuristic and analytical value, and Sraffa certainly made much use of this model in the development of his system of capitalistic prices. And that is precisely the point. Juxtaposed against this ideal type of unitary wage share prices (denoted p^0) are capitalistically-generated prices of production (denoted p^*), defined here as systems of prices when the wage share is less than unity and profits are uniformly distributed according to the value⁵ of the capital advanced; or stated precisely, the economic system in this (“non-ideal”) type operates under a less-than unitary wage share regime of functional distribution with equitable distribution of profits to the owners of capital according to the general rate of profits assessed to the value of capital advanced. But there is more. Sraffa’s unique contribution to the Classical paradigm involves the advancement of a second “ideal type” system of production, namely when the rate of profit is at its maximum value ($r = R$) and the share of wages zero (where prices are denoted p^R)

Such is the methodology that Sraffa employed. It is important to note that the analytical juxtapositioning of “early and rude” ($\omega = 1 : r = 0$) production relations with capitalist production proper ($\omega < 1 : r < R$) as well as maximum profit rate production relations ($\omega = 0 : r = R$) is a constant theme throughout Sraffa’s unpublished notes, and this clearly is a logical relation, not a historical one, as seen by Sraffa’s “mixing of the history” by repeated use of unitary wage share prices to analyze capitalistic relations of distribution.⁶ Sraffa refers to the logical construct of a unitary wage share price regime as the “value theory of labour”, namely the set of prices that result when the wage share set equal to unity:

D3/12/44/3

“21.2.1955

In the dust raised by the controversies on the Labour Theory of Value, a valuable ^{interesting} aspect has been overlooked, or what be called the Value theory of Labour...For, whatever disputes there may be about the determination of value by the quantity of labour, there can be no doubt (nobody doubts so far as I know) that the value of a commodity (its price for $r = 0$) determines (i.e. measures) the quantity of labour which directly or indirectly has entered into its production.”

Sraffa will use this construct repeatedly in his inquiries. The nice thing Sraffa found about what we will call “value theory of labour”-prices, or VTL-prices ($p^{VTL} = p^0$), was the ability to use this price form in the valuation of complex heterogeneous production systems, albeit imperfectly, to arrive at some very important properties concerning income distribution in such systems. And since for Sraffa the distribution problem was fundamentally a non-price phenomenon, initial conceptualization of these issues using these prices was completely justified as a first approximation at least.⁷ Formalizing, the “value theory of labour” for a basic economic system can be expressed as the following structure of prices:

$$p^0 Q = p^0 A + w^0 L \quad (1)$$

Where: $p^0 = (1 \times n)$ row vector of prices ascertained when $\omega = 1$ and $r = 0$

$w^0 =$ uniform wage per hour when $\omega = 1$ and $r = 0$; analogous to Pasinetti’s
“complete wage rate”

$Q = (n \times 1)$ column vector of gross outputs

$A = (n \times n)$ matrix of inter-industry means of production requirements

$L = (1 \times n)$ row vector of living labour requirements

System (1)⁸ is completely determined and the solution is unique. The wage rate w^0 represents the per unit productivity-qua-remuneration of each hour of productive labour when the wage share is unity (hence is equally productive as well as remunerative); here the labour bestowed by the productive workers is equal to the labour they command.⁹ This is the conceptual analogue of the “complete wage rate” that we find in Pasinetti’s (1977, p. 121) appendix on Marx’s transformation, although in Pasinetti’s inter-industry approach his would be a pure scalar while the above is expressed in remuneration per hour.¹⁰

Sraffa’s “Hypothesis” and the importance of the rate of exploitation

One of the upshots of Sraffa’s inquiries in the 1940s and that which directly led to the development of the Standard system was his conceptualization of the notion of a fundamental constancy in his economic system, what he referred to as his “Hypothesis” (see Gilibert 2003; de Vivo 2003; Bellofiore 2008, 2010; Kurz 2006; Gerhke and Kurz 2010; Kurz and Salvadori 2008, 2010). The main thrust of his Hypothesis was the idea that the value of social capital relative to social product remains constant in the face of changes in distribution and hence is also independent of price. As both Gilibert (2003, p. 36) and de Vivo (2003, p. 16) note with surprise, the idea that social capital relative to social product would be constant given changes in prices seems at first sight to be “most un-Sraffian”; Sraffa himself would in this period abandon the notion as it was originally conceived (he refers to the “disaster of the model” ; see especially Gilibert 2003, pp. 38-39), although the idea of invariance in the face of distributional changes would remain and eventually flower into the Standard commodity.

Bellofiore (2008) and Carter (2009) develop a line of analysis regarding the importance of Sraffa’s re-reading of Capital, Volume I in summer 1940 that underscores the importance the rate of

exploitation had for the manner in which Sraffa conceives of the problem of price-value deviations in the face of changes in income distribution. From notes written and inserted on an index card in his copy of *Capital*, Sraffa develops the notion of that the *source* of the deviation between value and price emanates from the extraction of unpaid labour, subsequently amassed in an abstract notion he would from 1942 clear through to December 1955 call the “pool of profits”. In 1940 Sraffa refers to this mass of extracted unpaid labour as “snow”:

“The greater the degree of exploitation in a society as a whole, the greater is the distortion (i.e. the divergence between values and prices). As, the greater the amount of snow fallen, the greater is the distortion of the surface of a piece of broken ground (i.e. divergence between the surface of the snow and that of the ground underneath; since the snow collects in the cavities)” (Sraffa 3731).

In Carter (2009) we document the development of Sraffa’s “snow” into the idea of the “pool of profits”. There it is shown that the “profits pool” notion became a fundamental driver in the development of what would later become surplus and deficit industries in Sraffa’s book. The basic idea here is simply that given different conditions of production as expressed as different ratios of labour to means of production, as the wage share falls the mass of unpaid labour gets distributed to the owners of capital as if it were doled out of a common fund, or “pool”, according to the value of the capital in the industry.

At the opposite end of the distribution, Sraffa conceives of prices that correspond to the set of prices at the zero wage rate and the maximum rate of profit, what Pasinetti calls the “pure capital theory of value” (1977, pp. 78-80). The equation system for this price system is:

$$p^R Q = p^R A(1 + R) \quad (2)$$

Where: $p^R = (1 \times n)$ row vector of prices ascertained when $\omega = 0$ and $r = R$

$R =$ maximum rate of profit

Here we find very clearly that Sraffa begins to conceive of the dual extremes of the distribution problematic (with zero profits and pure wages at one pole and pure profits and zero wages at the other) as both giving rise to an economically-relevant structure of prices. What will become important for Sraffa is that even if these prices are used, i.e. prices ascertained when the wage share is zero and profits are at their maximum, the ratio of fundamental constancy becomes no less evident and the maximum rate of profit is seen to “coincide” with the fundamental constancy of the “complete” value theory of labour price system.

3. Fundamental economic normalization

In moving from the subsistence economy to that of surplus, Sraffa in his book and in the Majorca Draft¹¹ at first juxtaposes the two systems in terms of the physical structure of production alone – i.e. shows by means of a simple two-commodity economy that the difference between the two systems lay in the increased physical output of the surplus system which uses the same conditions of production as the subsistence system. This can be clearly seen when the two physical structures of production used in his book are placed side-by-side:

[Insert Table 1]

Clearly the only difference in the two systems is the output of the wheat industry to the tune of 175 quarters of net output in the surplus production model. However, when it comes to formalizing the surplus model in terms of his equations, because of the adoption of the share approach to wages

Sraffa now feels it incumbent to introduce explicitly the quantum of living labour added. Quoting from the important Majorca Draft, we find the following:

“It therefore becomes necessary to ~~treat transfer wages from~~ treat wages, like the rate of profits, as one of the variables in the system. We shall therefore ^{have to} represent explicitly the quantity of labour employed in each industry – ~~instead of~~ in the place of the quantities of subsistence which we have hitherto rolled up with the raw material. We shall assume, on the lines of the class, that this labour is uniform or if not, that it has once and for all been reduced to uniformity by the ‘higgling and bargaining of the market’. We shall call L_a, L_b, \dots, L_n , the quantities of labour used respectively in the industry for producing commodity \underline{a} , etc; and we shall represent them as fractions of the total labour spent by the community in a year ^(which we take as unity) so that $L_a + \dots + L_n = 1$ (that is, we take as unit of measure of the L ’s the annual social labour)” (D3/12/52/6-7).

Sraffa accomplishes the same feat in paragraph 10 of his book (Sraffa 1960, p. 10). In both the Majorca Draft and published version we find Sraffa replacing the bundle of wage-goods with the quantum of living labour added, and this within the surplus-producing model. The physical structure of means of production remains constant for both the subsistence and the surplus-producing systems (witness Table 1 above), and this in effect results in the equating the living labour added with the net (surplus) product or national income. This equation is quite explicit in Sraffa’s book, where in paragraph 12 the value of the net product is also set equal to unity (see Bellofiore 2008, 2010). Adopting the same mathematical notation as Sraffa, we therefore have the following *economic* normalization of his system:

$$\underbrace{L_a + L_b + \dots + L_k}_{\substack{\text{Value added} \\ \text{by living labor} \\ \text{(Bestow)}}} \equiv 1 \equiv \underbrace{\left[A - (A_a + A_b + \dots + A_k) \right] p_a + \left[B - (B_a + B_b + \dots + B_k) \right] p_b + \dots + \left[K - (K_a + K_b + \dots + K_k) \right] p_k}_{\substack{\text{Value of net product} \\ \text{(Command)}}$$

We refer to the above equation/identity of the value-added by living labour to the value of the net product (both equal to unity) as the *fundamental economic normalization*. It is a normalization in the mathematical sense for certain, in that both are set equal to unity. However more importantly it is an *economic normalization* in that it renders clear the *economic hypothesis* that the quantity of new wealth, e.g. the net product, is the fruit of productive living labour. This idea is very clear in Sraffa's archival material, where he remarks that "the net product as a whole is always produced by L" (D3/12/29/18, dated December 1942). But the notion of equating the quantum of living labour with the value of the net product is not unique to Sraffa; indeed the entire open Leontief system and the "final sector" and "final demand" therein also is premised on this fundamental economic normalization (see Pasinetti, 1977, p. 61).¹²

The fundamental economic normalization and the Standard system

In the analysis that follows we remained confined strictly within an economic system already expressed in its Standard proportions and accordingly conceive the fundamental economic normalization within that context. This is done to illustrate our explicitly Marxian read of the analytical foundations in Sraffa's framework specifically as regards the question of exploitation and the logical distinction between extracted vs. distributed unpaid labour or profits. Whether or not this applies to an actual economic system remains to be developed and is outside the scope of the present analysis; although we are of the opinion that, yes, the conceptual framework indeed does apply. In this sense we are in agreement with Eatwell (1975, pp. 554-5) who argues that as long as in an actual economic system the wage is expressed in Standard commodity, the rate of exploitation derived via the Standard system (e.g. the ratio whose numerator and denominator is measured in units of Standard commodity) can be directly related to the actual rate of profit in the actual system

as a whole. Furthermore, it can be shown that within the Standard system both of Marx's aggregates, that (i) the sum of value equals the sum of prices of production, and (ii) the sum of surplus-value equals the sum of profits, are seen to hold. But again, in the present context we do not entertain the question of an actual economic system and instead stay within the safe waters of the Standard system. And in the Standard system, Pasinetti (1977, pp. 130-134) unequivocally demonstrates that whether we have ante-factum or post-factum wages, Marx's two aggregates do indeed hold and further that in this system the "value" rate of profit comes to coincide with the "price of production" rate of profit. Since this is a vitally important point in the analysis that follows, full quotation is on order. Note that in Pasinetti's formalization, \mathbf{Q}^* is the vector of Standard gross product when wages are paid post-factum and \mathbf{Q}^{**} is this selfsame vector when wages are conceived as ante-factum. When wages are paid ante factum the A-matrix is augmented to include the "subsistence wage" and since the A-matrix is so augmented, a "new" Standard system will arise out of the augmented A-matrix that Pasinetti denotes as $A^{(+)}$. Lastly the vector \mathbf{p} represents prices of production and the vector \mathbf{v} represents values:

"It is now interesting to check what happens to the three inequalities [i.e. Marx's two aggregates and the value-price rate of profit inequality]. First of all, total gross output, given by the vector \mathbf{Q}^* , is a scalar multiple of the standard net product, i.e. of the vector $(\mathbf{I} - \mathbf{A})\mathbf{Q}^*$. And, since $\mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{Q}^* = \mathbf{v}(\mathbf{I} - \mathbf{A})\mathbf{Q}^*$, it must also be true that $\mathbf{p}\mathbf{Q}^* = \mathbf{v}\mathbf{Q}^*$. Secondly, with the wages being paid in standard commodity, all that remains after the payment of wages – surplus value, if evaluated at 'values'; or profits, if evaluated at prices – will also consist of standard commodity and will therefore be a scalar multiple of the net product. This means that total surplus value will be equal to total profits. Finally, the rate of profit, by being a ratio between two scalar multiples of the same standard net product, will remain the same, whether evaluated at 'values' or prices. *In other words, all three*

inequalities...become equalities in the present case... [E]ven with the [ante factum wages] normalization, the three inequalities ...become equalities in this new standard system. Total surplus value and total profits are equal to each other owing to $\mathbf{p}(\mathbf{I} - \mathbf{A}^{(+)})\mathbf{Q}^{**} = \mathbf{v}(\mathbf{I} - \mathbf{A}^{(+)})\mathbf{Q}^{**}$. Moreover, $\mathbf{p}\mathbf{Q}^{**} = \mathbf{v}\mathbf{Q}^{**}$, because each side of this equality is the same scalar multiple of the corresponding term [in the surplus value = total profits equation]. Finally, as in all standard systems, the rate of profit is a physical notion and is therefore independent of both ‘values’ and prices” (Pasinetti, 1977, pp. 133-134; emphasis added).

Hence the Standard commodity and the Standard system generally renders perfectly consistent the value added by living labour as equated with the value of the net product; that is to say, here there is no “leakage” of value as is the more general case of an economic system expressed in its actual (read non-Standard) proportions. In this manner we bring in our Marxian reading yet another interpretation to the idea of the Standard commodity as a “purely auxiliary construction” (Sraffa 1960, p. 31) in that it represents the anchor which renders transparent the *origin* of profit as extracted unpaid labour subsequently distributed to capitalists via the general rate of profit – i.e. it renders transparent the extraction-distribution nexus. And within the Standard system, the above normalization (identity) is assumed to hold throughout, i.e. at all regimes of functional income distribution. It represents the cement, so to speak, of our system of generalized commodity production. If we are able to translate into the language and conceptualization of Classical theory, the identity states that the addition of new value added on the labour-side – understood here as value *bestowed* – must be equal (proportional?) to the value of the new product resulting from this productive labouring activity – understood here as value *commanded*. Hence bestow and command as conceptual categories are fundamentally related to each other prior to the introduction of the distribution problematic; furthermore the entire problem of distribution can be understood in this

framework as the existence of fundamental asymmetries between bestow and command at the level both of “factor” inputs as well as commodities. It is from this normalization that the Marxian concept of exploitation can be best understood (again this echoes Eatwell 1975). This has to do with the Classical notion that the labour bestowed on the productivity side of the identity comes to equal the resulting new wealth commanded on the use-value side of the identity. It is to this question of bestow and command that we now turn.

Bestow and command

The bestow relation belongs to the realm of production. Value bestowed in production is synonymous to the value embodied, directly and indirectly, in the production of a commodity. This is the common definition of (labour) values whose properties are well known. The command relation belongs to the realm of exchange. But the definition of “command” is not as cut-and-dry, and one finds in the literature a variety of different applications and definitions of what is meant by “labour commanded”.

That the labour command concept is subject to different definitions is the perspective taken in a recent paper by Naldi (2003, pp. 550-554) where he gives two interpretations of this concept:

1. Labour commanded = quantity of other people’s labour an exchange with one’s own brings forth
2. Labour commanded = quantity of output received through the wage bargain

The first interpretation considers the command relation at the level of the generalized exchange of commodities. One’s own labour here is embodied in a commodity, and this is taken to market in exchange for another commodity, itself the result of someone else’s labour. In generalized commodity production the products of labour exchange at ratios determined by their embodied

(bestowed) labour; or in more Marxian terminology, the materialized labour in a commodity product commands a quantum of materialized labour of equal value embodied in another commodity product, or “money” which is universal generalized command. Here the command concept is brought into equality with the bestowed conception, and in *Theories of Surplus Value* Marx referred to this as the law of value. The second interpretation is the standard definition of labour commanded, namely the quantity of output received through the wage bargain, defined as the (net) “money” price of consumption goods over the “money” wage: $\left(\frac{p_Y}{w}\right)$, where p_Y = index of price of consumption goods (or net product) and w = money wage. It is our opinion that both of these definitions of the labour commanded concept are correct in their own contexts, and a unified theory of command and bestow can be developed that brings some order in the various definitions of the former and a synthesis of it to the latter. In this way we can begin to close this still open issue in the Classical theory of value and distribution. To see this consider the following price structure for a hypothetical basic economic system with post-factum wage payment:

$$p^* Q = p^* A (1 + r^*) + w^* L \quad (3)$$

Where: p^* = $(1 \times n)$ row vector of prices of production when $1 > \omega > 0$

w^* = uniform wage rate per unit labour when $1 > \omega > 0$

r^* = uniform rate of profit when $1 > \omega > 0$

The remuneration to the “factors” of production, namely the capital-cost $(p^* A)$ and labour (L) , is given by the net product equation:

$$p^* Y_{np} = r^* p^* A + w^* L \quad (4)$$

Where: $Y_{np} = (n \times 1)$ column vector of net outputs

In terms of the framework we are developing, equation (4) represents the exhaustion of net product as factor remuneration on the “command-side” or exchange side of the ledger. Hence the fundamental normalization identity becomes:

$$\underbrace{w^0 L}_{\text{Value bestowed}} \equiv \underbrace{p^* Y_{np}}_{\text{Value}} = \underbrace{r^* p^* A}_{\text{Value commanded}} + w^* L \quad (5)$$

Value bestowed
by productive
living labor

Value
commanded by
the net product

Value commanded
by “factor” inputs

One of the significant features of the characterization of the economic normalization in equation (5) is the extension of the concept of command **to include not only wages but also** net output and profits. We find command both on the output side (net product) as well as command on the input side (“factor” remuneration). Regarding the latter, wage remuneration will indeed command a certain quantum of labour, *but so too will profit remuneration*. This is very much in the spirit of the fifth measure of value that Malthus advanced, namely “the constant labour that produces the wages and profits in a commodity” (*Works XI*, p. 107; on this see Glyn 2003, Carter 2010b).

The capital value is defined as the price of the means of production inputs, here referred to as $(p A)$, hence the value embodied (bestowed) in the means of production represents this initial capital value. The *command* of this capital value – i.e. the quantity of value this capital can exchange for in the market - is a varying quantity depending on the level of the wage share. At a unitary wage share, the quantity of value a capital commands is equal to its value bestowed in production and nothing more. However, as the wage share falls, by virtue of capitalistic relations of private property, capital commands more value than its cost. The opposite happens in the case of labour. At the

unitary wage share labour too commands the value it bestows. However for this “factor”, as the wage share falls, the command of the product of labour falls as well. Thus workers command, through their remunerated wages, a quantity of labour bestowed in the commodities they purchase with these wages that is lower than the living labour they perform in the process of production. This is the fundamental asymmetry in income distribution that we read from our Marxian approach to the analytical foundations in Sraffa’s framework.

Thus we arrive at the fundamental asymmetry in factor command associated with capitalistic relations of distribution; and it is this asymmetry that caused Marx to exclaim that in capitalist commodity production, the law of value (read bestow = command) breaks down.¹³ The asymmetries associated with the command of factors was also identified by Malthus in his advancement of the labour commanded standard over Ricardo’s labour embodied one.¹⁴ The asymmetries in factor command in more-than-one commodity models come to express themselves at the level of commodities in terms of asymmetries in prices.

4. Asymmetries in factors and asymmetries in commodities

In this section we develop the implications of the above for Sraffa’s general theory of price. Capitalistic production necessarily entails the emergence (creation) of a surplus, and consistent with our Marxian interpretation of Sraffa’s foundations, this surplus is by definition surplus-value, i.e. unpaid labour. It results from changes in the distribution parameter such that the fall in the wage share causes asymmetric and off-setting command of each of the factors of production: capital commands a greater value than its cost, labour commands less value than its productivity. We find here very clearly the Marxian theory of exploitation. Because we remain within the Standard system, value is conserved in that the mass of surplus value extracted will equal the mass of profits remunerated. In his unpublished notes, over a period we have documented lasting 14 years, Sraffa

conceived of this process in terms a “pool of profits” (see Carter 2009). This “pool of profits” we argue became for Sraffa the conceptual equivalent of the [notion](#) of extracted unpaid labour.

We have already seen that the mass of extracted unpaid labour is fundamentally related to the distributional problem in capitalistically generated prices as keenly noted by Sraffa in marginal notes he penned on his English edition of *Capital*, Volume I, that he read while interned on the Isle of Man in 1940. Here the distribution problem is clearly related to the question of extraction and distribution of a mass of unpaid labour. What was “snow” in 1940 became by November 1942 the “pool of profits.” The latter appears in an important document entitled simply

“Exploitation’, cont.”

“12.11.42

‘Exploitation’ cont.

As wages decrease by a given proportion the total of profits, for society as a whole, increases by the same amount. But this is by no means true for any individual industry taken as separately: the increase in the total of profits, in any one industry, may be e.g. smaller than the decrease in the total wages paid in that industry. Where does the difference go? It goes, of course, to industries where the profits increase more than by the fall in their wages. And the adjustment is brought about by a fall in the price of the commodities produced by the former industry and a rise of those produced by the latter.

Thus, the proceeds of a reduction of wages, don’t simply *pass from the pockets of the workers* to those of their employer. They go, as it were, into a social pool ^{of profits}, to which each industry contributed in proportion to the importance of its particular variable capital; and which is

then shared out, among the various industries,^{each receiving} in proportion to their particular total capital (variable and constant)” (D3/12/17/3; italicized emphasis added).

Here Sraffa seems to us to clearly refer to the process of extraction of unpaid labour, seen especially with the idea of the proceeds of the wage reduction “pass[ing] from the pockets of the workers”.

We find the use of the metaphor “pocket” very instructive and take it as an indication of property ownership – if something is in your pocket, ostensibly you own it. And it is also instructive that the “proceeds” that pass from the pockets of workers to those of capitalists given a reduction in wages do so without an equivalent counter-exchange; wages are reduced whereas the quantum of living labour remains constant, and the coffers of profits swell, to be distributed to each capitalist *not* in proportion to the quantum of “proceeds” workers in each industry “pass”, but rather according to the value of the capital advanced. In our explicitly Marxian read of this, extracted unpaid labour is amassed in the guise of a “pool of profits” only to be doled out to each aliquot capital according to the value of that capital advanced. Marx in *Theories of Surplus Value* and in a letter to Engels in 1868 refers to a similar reckoning process of distribution of extracted profits to each aliquot capital according to the general rate of profits as “capitalistic communism”.¹⁵ Sraffa would use the concept of the “pool of profits” from this point up through February 1956, where for reasons he never specifies he simply abandons the concept altogether and instead begins to utilize the surplus and deficit industry concept to frame the question of value transfers across industries.

In our Marxian read of Sraffa’s foundations, we accept the notion that there is indeed a mass of extracted unpaid labour, and that the asymmetries in factor command express themselves as asymmetries in the capitalistically generated prices. Consider the respective price forms for three regimes of functional income distribution: (i) when the wage share is unity; (ii) when the wage share is between one and zero; and (iii) when the wage share is zero:

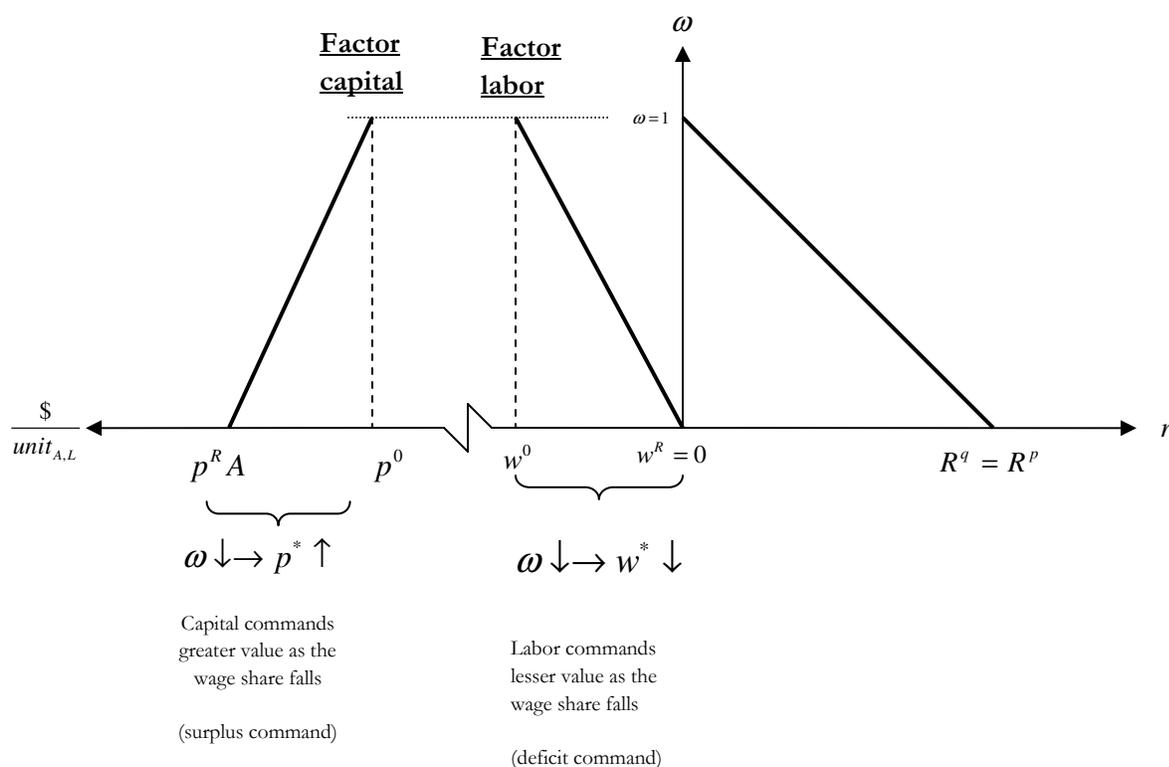
$$\omega = 1 : r = 0 : p^0 Q = p^0 A (1 + r^0) + w^0 L = p^0 A + \{w^0 L\} \quad (\text{pure labour command})$$

$$\omega < 1 : r < R : p^* Q = p^* A (1 + r^*) + w^* L = p^* A + \{p^* A r^* + w^* L\}$$

$$\omega = 0 : r = R : p^R Q = p^R A (1 + r^R) + w^R L = p^R A + \{p^R A R\} \quad (\text{pure capital command})$$

The terms in the curly brackets represents the distribution of the net product across the different distribution regimes. We have defined command as the quantity of value a (given) sum of value bestowed is able to exchange for in the market. As the wage share falls, the command of capital will include its own cost plus the “overplus” associated with any given level of income distribution; alternatively, the command of labour will always be of a value less than its own productivity, the minimum value of which is zero. The case when the wage share is unity corresponds to pure labour command in the sense that the net product is distributed as complete wages, and the case when the wage share is zero corresponds to pure capital command in the sense that the net product is distributed as complete profits. Here the inverse wage-profit relation is expressed in terms of factor commands.

Figure 1: Value transfers of factors of production and the wage share profit rate schedule



Quadrant I shows the wage-profit tradeoff according to the familiar proportionality Standard between the wage share and the rate of profit:

$$r = R^q (1 - \omega) = R^p (1 - \omega).$$

In this is equation we have followed Sraffa from notes of December 1955 (D3/12/57/23) and give two different expressions of the ratio R . The ratio R^p is the maximum rate of profit ascertained when the system is solved for zero wage share prices. The ratio R^q is the Standard ratio, a uniform physicalist ratio ascertained by virtue of the q-system of quantity multipliers from the Standard system. It is interesting to note that explicitly equating the “value” relation via R^p with the “physicalist” relation via R^q is precisely how Eatwell (1975 p. 548, equation 5) expresses the same

sentiment. Quadrant II shows the remuneration of factors. As the wage share decreases, quantities of value “pass from the pockets of the workers” into a social profits pool from which it is then distributed to the “pockets of the capitalists”. If we may borrow the terminology Sraffa employs, the factor capital experiences *surplus command* and the factor labour experiences *deficit command* as the wage share falls. These surplus and deficit commands of the factors will express themselves in surplus and deficit industries when expressed at the level of commodities.

Surplus and deficit industries

The heterogeneous physical structure of production is rendered commensurate at prices associated with the various levels of the wage share. We have considered three different regimes of the wage share, and hence three sets of equation for each industry. Having defined the price form, we use that associated with a unitary wage share (e.g. VTL-prices) to ascertain the value ratio of labour to means of production for each industry, which following Sraffa we call the *labour to means of production ratio*, or LMP ratio. The respective industry’s LMP ratio will be ascertained as follows:

$$LMP_i = \frac{w^0 L_i}{\underbrace{(p_i^0 A_{ii} + p_j^0 A_{ji} + \dots + p_n^0 A_{ni})}_{\text{Industry } i \text{ is labor intensive}}} > \frac{w^0 L_j}{\underbrace{(p_i^0 A_{ij} + p_j^0 A_{jj} + \dots + p_n^0 A_{nj})}_{\text{Industry } j \text{ is capital intensive}}} = LMP_j$$

In the multi-commodity case by definition the conditions of production differ across industries. In our model we assume that industry i is labour intensive and industry j capital intensive as reflected by a higher LMP ratio in industry i vs. industry j .

Sraffa notes that there will be a “watershed” ratio marking a significant balance in the economic system. It can be shown that this watershed ratio is the average LMP ratio in the Standard economic

system, denoted LMP^* , and that the following property exists between the different LMP ratios in our system:

$$LMP_{LI} > LMP^* > LMP_{KI}$$

Note that we have replaced subscripts i and j with LI and KI, indicating labour-intensive and capital-intensive industries respectively. Consider the remarkable equality in the various ratios across the different aspects of our economic system when expressed in its Standard proportions:

$$R^q = LMP^* = R^p$$

Here the physical structure of production (as expressed by the Standard ratio R^q) comes to coincide with the value structure of production (as expressed by LMP^*) as well as relations of distribution (as expressed by the maximum rate of profits R^p).¹⁶

We remind the reader that the present analysis is conducted strictly within the confines (or “safe waters”) of the Standard system. We have also seen, as Pasinetti (1977, pp. 133-4) has unequivocally demonstrated, that in the Standard system the sums of value and price and surplus value and profit coincide. Hence here the effect of changes in price when we change distribution of the value-added (or what is the same, increase exploitation of workers) can be unequivocally isolated according to the proportions of labour to means of production of the respective industry relative to the Standard ratio. Hence in the Standard system, the anarchy of price movements both possible and likely in an actual (non-Standard) system such that “the relative price of two products may move, with the fall of wages, in opposite direction to what might have been expected on the basis of their respective ‘proportions’” (Sraffa 1960, p. 15) no longer vexes the economic system. The construction of the Standard system *ipso facto* ensures complete price-redress vertically-integrated throughout the

economic system (the same result occurs when the system is subject to the reduction to quantities of dated labour). To use the language invoked by Pasinetti (1977, pp. 82-83), when an economic system is expressed in its Standard proportions and all prices (including wages) are measured in composite Standard commodity, the “first-order” capital intensity effect is the only one that matters when income distribution changes (exploitation increases), the “second-order” price effect being rendered moot. This can be seen when the price of commodity j is expressed in terms of the Standard commodity, the relative price of which (when the “incomplete wage rate” = $w = 1$) will be given by:

$$\frac{p_j}{P_{STND}} = \frac{l_j + (1+r) \sum_{i=1}^n a_{ij} p_i}{l_{STND} + (1+r) \sum_{i=1}^n a_{iSTND} p_i}$$

where in this equation all physical magnitudes are *not* “given quantities” but rather per unit gross output (i.e. l_i = direct labour of industry i per unit output industry i and a_{ij} = input good i necessary to produce one unit of good j).

Pasinetti (1977, p. 83) shows that when the relative price equation above is differentiated with respect to the rate of profit, then in the neighborhood of r the total differential can be decomposed into (i) the “first-order” capital-intensity effect, and (ii) the “second-order” price-effect :

$$\frac{d}{dr} \left(\frac{p_j}{P_{STND}} \right) = \underbrace{\left[P_{STND} \sum_{i=1}^n a_{ij} p_i - p_j \sum_{i=1}^n a_{iSTND} p_i \right]}_{\substack{\text{Capital-intensity effect} \neq 0 \\ \text{(first-order effect)}}} + (1+r) \underbrace{\left[P_{STND} \sum_{i=1}^n a_{ij} \frac{dp_i}{dr} - p_j \sum_{i=1}^n a_{iSTND} \frac{dp_i}{dr} \right]}_{\substack{\text{Price effect} = 0 \\ \text{(second-order effect)}}}.$$

The device of the Standard commodity, and hence the Standard system, is constructed such that the price effect with changes in income distribution within the second bracket is offsetting and thus equal to zero. Thus changes in distribution, which in our Marxian reading necessarily involves the extraction of unpaid labour, affect the price system only according to the intensity of production relative to that of the Standard or watershed.

The key to the determination of the direction of value transfers, whether and how we can unequivocally say an industry is in surplus or deficit, requires that we develop the implications on the system of prices with a wage-reduction. What exactly happens in the various industries of the Standard system when the wage is reduced? How do the proceeds from a wage reduction affect the “pockets of the workers” in the various industries? To answer these questions, we need to consider the value bestowed relation in the normalization identity at the various levels of the wage share. And here it is convenient to consider the respective distributive shares, the wage share (ω) and the profit share ($1 - \omega$), each assessed against the net productivity of labour, expressed by the “complete” wage (w^0). This gives rise to two distinct expressions of “incomplete” wage-forms. On the one side, the wage relation presents the command of output workers receive at the various levels of the wage share; hence the wage becomes the *relative wage*, defined as the wage share times the net productivity of labour:

$$w_{(+)}^* = (\omega)(w^0).$$

On the other side, proceeds are “passed from the pockets of the workers” in the sense that at each regime of remuneration as wages, there exists an associated extraction of profits. Here the profit share is multiplied by the productivity of labour. This is an extractive process, the opposite of the remunerative process involved on the wage-side. Hence we may be justified as conceiving this

“price form” as an extraction per hour, the opposite-wage-rate paid, or what we may call the *relative anti-wage*, denoted $(w_{(-)}^*)$:

$$w_{(-)}^* = (1 - \omega)(w^0).$$

Pasinetti (1977, p. 124) arrives at a very similar idea, referring this construct as the “unpaid wage”.¹⁷

Thus the following fundamental identity exists between the two relative wage forms:

$$w^0 = [(\omega)(w^0) + (1 - \omega)(w^0)] = (w_{(+)}^* + w_{(-)}^*)$$

With development of the relative wage and the relative anti-wage, the one a remunerative relation, the other an extractive relation, we can ascertain with certainty how with a reduction in the wage share the proceeds “pass from the pockets” of each industry’s workers to each industry’s capitalists:

$$\begin{aligned} LI : w^0 L_{LI} &= w_{(+)}^* L_{LI} + w_{(-)}^* L_{LI} \\ KI : w^0 L_{KI} &= w_{(+)}^* L_{KI} + w_{(-)}^* L_{KI} \end{aligned}$$

The LHS of each equations shows the valued bestowed by living labour for each industry; the RHS shows the division of the value-added per industry into a paid portion $(w_{(+)}^* L_i)$ and an unpaid portion $(w_{(-)}^* L_i)$. We refer to the latter as *extracted unpaid labour* or *extracted profits*, and the sum of the extracted profits for the system as a whole is Sraffa’s pool of profits:

$$\text{Pool of profits} = (w_{(-)}^* L_{LI} + w_{(-)}^* L_{KI}) = \Sigma \text{ extracted unpaid labour} = \Sigma \text{ proceeds passed from workers pockets}$$

It is our contention that this reveals, along Marxian lines, the *origin* of surplus value based in the exploitation of labour, and that further in order to so reveal this, the abstract construction of the Standard system is both necessary and sufficient.¹⁸ It is in this manner the unequivocal “surpluses”

and “deficits” that arise through the extraction-distribution nexus can be approximated and understood.

The unpaid labour extracted by each industry according to the relative anti-wage concept allows for the development of the individual rates of profit for each industry, defined as the extracted unpaid labour of that industry divided by the value of the means of production of that industry.

$$r_{LI} = \frac{w_{(-)}^* L_{LI}}{\sum p^* A_{LI}} > \frac{w_{(-)}^* L_{KI}}{\sum p^* A_{KI}} = r_{KI}$$

$$w_{(-)}^* L_{LI} = r_{LI} \sum p^* A_{LI} > w_{(-)}^* L_{KI} = r_{KI} \sum p^* A_{KI}$$

The denominators of the equations in the top row represent the sum of prices of means of production multiplied by these input requirements. Labour intensive industries will have a greater individual rate of profit than capital intensive industries, and because we are in the Standard system we know that this inequality persists unequivocally across all levels of the wage share, including the zero wage share; hence:

$$R_{LI}^p > R_{KI}^p$$

The general rate of profit can be expressed as the arithmetic average according to the following formula:

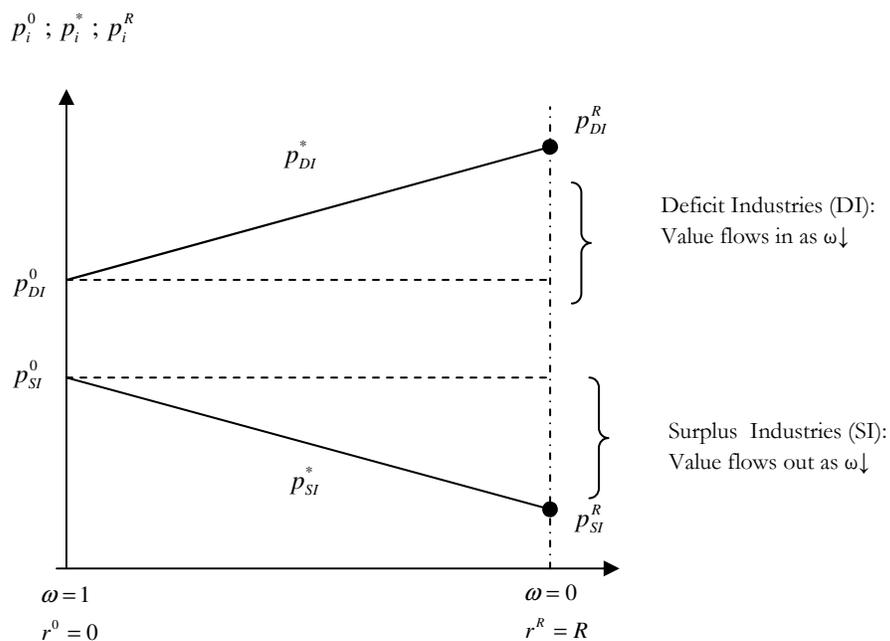
$$r^* = \frac{\sum_{i=LI}^{KI} (w_{(-)}^*)_i L_i}{\sum_{i=LI}^{KI} p^* A_i}$$

It will be at this general rate (r^*) that capitalists in each industry will be remunerated. And here we express the distinction between extracted unpaid labour or profits (Z^0) and distributed unpaid labour or profits (Z^*), each per industry, as shown in the following table:

[Insert Table 2]

As the wage share falls, labour intensive industries will exhibit a *surplus* in the profits extracted when compared to the profits remunerated (distributed) to the capitalists of that industry; alternatively capital intensive industries will exhibit a *deficit* in the profits extracted when compared to the profits remunerated (distributed) to the capitalists of that industry. Sraffa refers to the labour intensive industries as *surplus industries* and capital intensive industries as *deficit industries*. We note that in the Standard system these value transfers are unequivocal because the price-effect here is moot. Notice as the wage share falls surplus industries command less value than is bestowed in the production of the commodities of that industry, and deficit industries command a greater value than that bestowed in the production of the commodities of that industry.¹⁹ We can express these relations in the following graph:

Figure 2: Surplus and Deficit Industries



Sraffa on at least two different occasions separated by 14 years drew a very similar graph, which he called “spectrogram” (see D3/12/17/2, September 1942; and D3/12/56/32, February 1956). It is especially of interest to highlight that the September 1942 notes where this graph appears is entitled “Exploitation”:

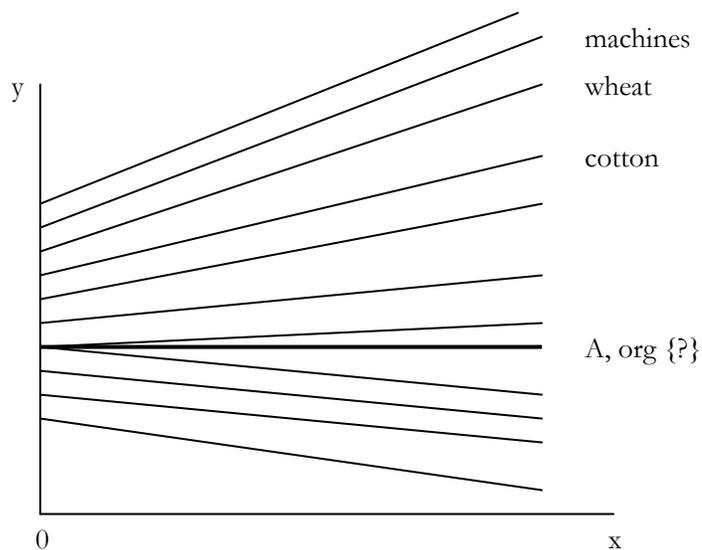
D3/12/17/2

“20.9.42

‘Exploitation’

Having built up the system, make it move. Begin by moving w , by steps, to its maximum position. All the prices of individual commodities will move, some rising, some falling.

Represent all this by a single diagram. On the ordinate represent prices so that there is a curve for each commodity: arrange the commodities, from top to bottom, in order of decreasing organic composition, so that, with a fall of w , the top commodities will fall, the middle one unchanged, and the bottom one rise. The point Oy represents ‘values’.



On the abscissa represent $1 - w$, or s/v , or some other function of the proportions in which the product is distributed (!); choosing the function that gives the simplest curves, preferably the one (if available) that gives straight lines.

w increases equally in all industries. We notice that in low composition industries, w , as it increases, absorbs more than the total profits previously made in that industry; when w reaches its maximum (at the point 0) in each of the industries below the middle, w has increased more than the profits have decreased in the same industry. (on the other hand in industries above the middle, w rises by less than profits fall”).

And also from notes written in 1956:

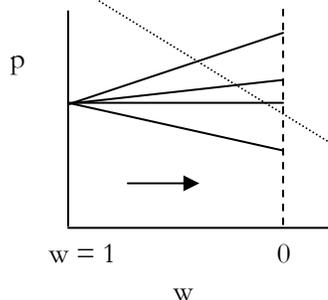
D3/12/59/32

12.2.56

“Under these circumstances, as we reduce the wage the prices of commodities would depend exclusively and in a simple manner from the proportions by which they were produced. The price of each commodity would uniformly rise, or uniformly fall, throughout the interval, the degree of the rise and fall depending exclusively on the proportion of labour to means by which it was produced.

We could arrange the various products in the order of the degree of price variation beginning with at one extreme with the one whose rise was steepest and ending with the one which fell most sharply.

“In the following diagram the unit of each commodity is supposed to be chosen of such rise as to be equal in value:

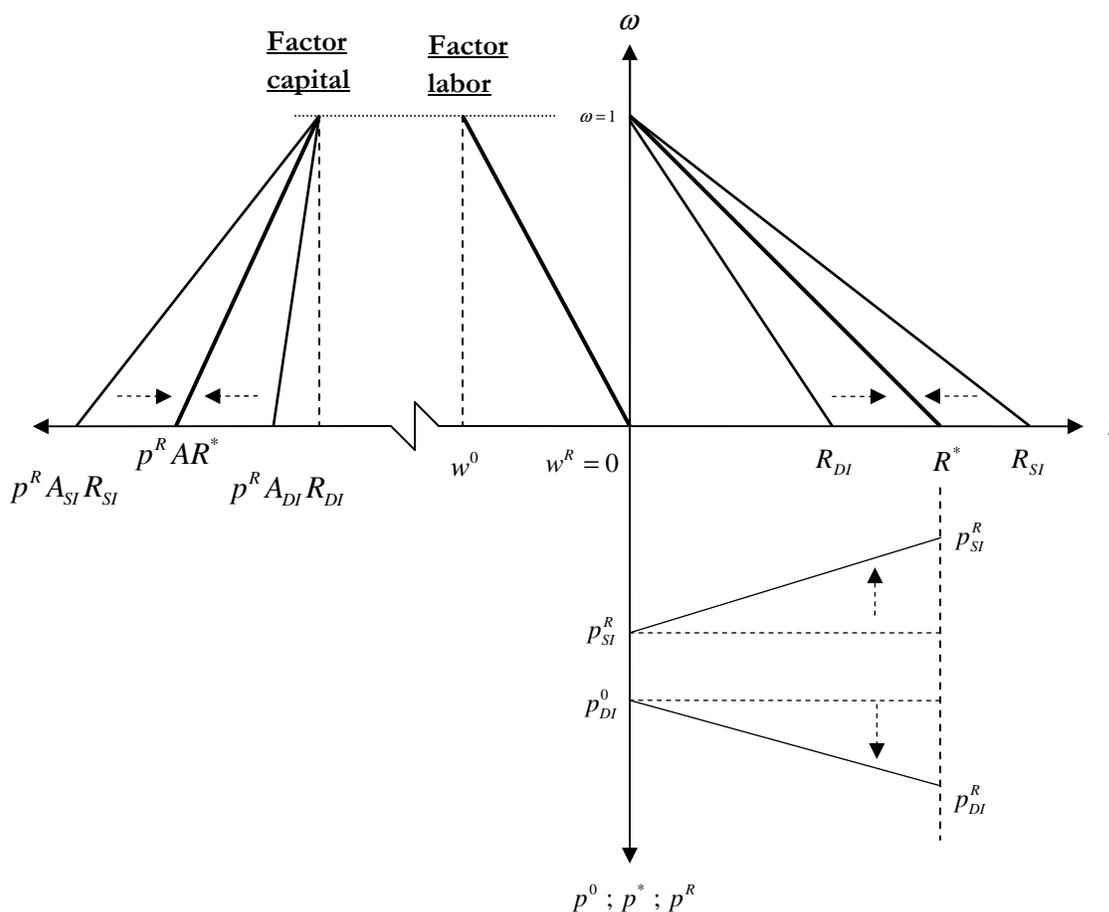


“In the middle there is just room for one product which has no tendencies either to rise or to fall: since the standard of prices is the aggregate means of production this implies that it is produced by such proportions of labour to means of production that there is within the industry producing it an exact balance at all steps in the variation between deduction from wages and addition to profits.”

It is very interesting to note that Sraffa crossed the diagram out, and speaks to the notion that as he got closer to publication, Sraffa somewhere decided to scale back the explicitly Marxian character of his inquiries.²⁰

As a final step, let us juxtapose the diagram showing the command of factors and the wage-profit trade-off with that of the value transfers from surplus and deficit industries. This is shown in the following figure:

Figure 3: Value transfers of factors of production, the wage share profit rate schedule and Surplus and Deficit Industries (2 commodity world)



As before Quadrant II represents the command of the factors. Workers in labour intensive industries will “pass from their pockets” a greater mass of unpaid labour than worker from capital-intensive industries, although both will be operating under a uniform rate of exploitation. The schedules of the respective maximum rates of profit for the different industries (R_{SI} and R_{DI}) are shown, and the process of “pooling” the unpaid labour is shown by the arrows of gravitation towards the general rate of profits. The wage-profit schedules in Quadrant I also reflect this gravitational process. It is important to point out that the asymmetries on the capital side of the ledger express themselves in convergence towards a common center of gravity around the general rate of profits (r^*). The asymmetries in factor command will express themselves as asymmetries in prices of production.

5. Conclusion

This paper has attempted to demonstrate that Marx's concept of exploitation was both an instrumental mechanism in the way Sraffa conceptualized his economic system as well as an underlying theme he retained in the pages of his book. We have shown that exploitation framed the very manner in which Sraffa conceived of price-value deviations, to the extent that, as evidenced in his "snow" metaphor for the notes on *Capital*, Volume I, the very possibility of such deviations arises *causally* from the existence of unpaid labour, and that these deviations result from the distribution of the extracted "pool of profits" to owners of capital according to the general rate of profit assessed to the latter's different value of capital advanced. We argue that this becomes clear once the economic system is expressed in its Standard proportions. Hence we give another perspective on Sraffa's observation that "particular proportions, such as the Standard ones, may give transparency and render visible what was hidden, but they cannot alter its mathematical properties" (Sraffa, 1960 p. 23). The "mathematical property" that this paper has attempted to show is the idea that within the Standard system, Marx's two aggregates hold, and further that this evidences the *origin* of profits as emanating from extracted unpaid labour.

All of this translates into a nexus connecting the productivity of labour, the offsetting remuneration to the competing "factors", and the resultant price-value deviations. This distribution-price transfer nexus for our system was illustrated in Figure 3. There it is clear that as regards the first-order condition (Pasinetti's "capital-intensity effect"), prices relate to the process whereby different quantities of unpaid labour extracted at the industry level must be equitably distributed to the owners of those industries according to the supply price of their capital; this we referred to as the process of "pooling" of profits. The inter-capital *convergent (gravitational)* processes in the two upper quadrants reflect the processes involved in the formation of a (competitive) general rate of profit to

redress this problem. This is interestingly juxtaposed to the *divergent* process at work in terms of the price system, seen in quadrant IV. There prices of production push *away* from the center of gravity (e.g. VIL- *qua* labour-values) as the wage share falls. This seeming contradictory movement, convergence in one sense, divergence in another, has led to a tremendous amount of confusion in disquisitions on value and distribution in all types of economic theory. And this latter was, we contend, what Sraffa was attempting to demonstrate.

Table 1: Production for subsistence and production with a surplus numeric examples in Sraffa (1960)

Production for Subsistence (Sraffa 1960, Ch. I, p. 3)	Production with a Surplus (Sraffa 1960, Ch. II, p. 7)
$\begin{pmatrix} 280 \text{ qr. wheat} & : & 12 \text{ t. iron} & \rightarrow & 400 \text{ qr. wheat} \\ 120 \text{ qr. wheat} & : & 8 \text{ t. iron} & \rightarrow & 20 \text{ t. iron} \end{pmatrix}$ $\Sigma = (400 \text{ qr. wheat} : 20 \text{ t. iron})$	$\begin{pmatrix} 280 \text{ qr. wheat} & : & 12 \text{ t. iron} & \rightarrow & 575 \text{ qr. wheat} \\ 120 \text{ qr. wheat} & : & 8 \text{ t. iron} & \rightarrow & 20 \text{ t. iron} \end{pmatrix}$ $\Sigma = (400 \text{ qr. wheat} : 20 \text{ t. iron})$

Table 2: Extracted and distributed profits

Extracted profits	Distributed profits
$w_{(-)}^* L_{LI} = r_{LI} \sum p^* A_{LI} = Z_{LI}^0$	$Z_{LI}^* = r^* \sum p^* A_{LI}$
$w_{(-)}^* L_{KI} = r_{KI} \sum p^* A_{KI} = Z_{KI}^0$	$Z_{KI}^* = r^* \sum p^* A_{KI}$

¹ Scott Carter, The University of Tulsa (scott-carter@utulsa.edu). Please do not quote without permission as Sraffa's archival material is copyright by his Literary Executor, Professor Garegnani.

² This idea that Sraffa came to many of the same conclusion as Marx especially in the early 1940s is developed in varying degrees in de Vivo (2003), Gilibert (2003, 2006) Carter (2009), Bellofiore (2008, 2010), Gehrke and Kurz (2006), Kurz (2006), and Kurz and Salvadori (2010).

³ Both of these aspects of Sraffa regarding income distribution have proved quite puzzling to neoclassical economists since the publication of his book. This is perhaps best captured in Melvin Reder's (1961) review of Sraffa (1960) in the *American Economic Review*, who, quite noticeably aghast, remarks that:

“Sraffa offers no theory of distribution; so far as this book is concerned, the wage share may be represented by any real number between zero and one, and the profit share by one minus the wage share” (p. 691)

Clearly by a “theory” of income distribution Reder infers as necessary a *behavioural* functional relationship between the “supplies” of “factor inputs” and the attendant “rewards” these factors, at the margin of constrained choice, receive.

⁴ Adam Smith of course was the first to utilize this method of juxtaposing price formation in capitalist vs. non-capitalists systems of commodity production as seen in the movement from simple labour value in the “early and rude state” to an added-up value theory when “stocks had accumulated into the hands of particular persons...”; Ricardo and Malthus also used this methodology in developing their respective theories of value, but discussion on the “early stages of society” did not have as prominent a role in their published work as it did in their correspondence. Marx of course adopted this approach explicitly, devoting the first three chapters of *Capital*, Volume I to non-capitalist commodity production. But we cannot leave out some neoclassical theorists, especially the Austrian approach, that also conceived of an ideal state along the lines of pure-labour remuneration as seen in the interpretation of the possibility of finite reduction to dated quantities of labour “far enough back” to reveal pure unassisted labour as the only (“original”) primary factor of production. Sraffa of course critiqued this approach to the reduction, arguing especially in his Papers emphatically that no matter how “far back” one goes there will always be a “commodity residue” that must be reckoned with; indeed this was the basis for the maximum rate of profit concept.

⁵ A word need be said here about the term “value”. Throughout the remainder of the present analysis we shall adopt Sraffa’s own definition of “value” that appears in his important “terminological note” in paragraph 7 of his book, namely as expressing the “exchange ratio [which] depends as much on the *use* that is made of [a basic product] in the production of other basic commodities as on the extent to which those commodities enter its own production” (Sraffa 1960, pp. 8-9). This is irrespective of the particular *numeraire* adopted, although as discussed below the particular *numeraire* adopted does matter in the determination of the size of the pie in non-Standard systems. It is of great interest that in the Majorca Draft Sraffa speaks to the question of “value” *not* in the surplus producing system as is the case for the “terminological note” in his book, but rather does so immediately in the subsistence producing system. The phrase “terminological note” is how Sraffa himself characterizes this important paragraph 7 in the table of contents of his book (see Sraffa 1960, p. *ix*). In general it is very useful to go through the sections headings in Sraffa’s table of contents in that it can give a concise indication of the point of each section, especially, as Ajit Sinha (2010, p. 277) has said, Sraffa is an exercise in “minimalist art”.

⁶ Now of course no such “early and rude” society ever really existed, and this is related to the question of the “historical” labour theory of value, a point made quite convincingly in a recent contribution by Kurz and Salvadori (2010). They argue that beginning with Smith, and including the likes of Torrens and Engels, it “had been contended that the labour theory of value holds in ‘primitive’ societies”, but that “apparently Sraffa was not convinced” (Kurz and Salvadori, 2010, p. 198). They cite very interesting evidence that Sraffa seriously considered the merits of this question, studying “books on economic history, anthropology, and ethnology and noted *inter alia* passages dealing with the historical interpretation of the theory” (Id.). It is ultimately shown that Sraffa came down against the idea of a “historical” approach to the labour theory, and we concur with this assessment. For Sraffa the question turned primarily on the analytical distinctions between the two separate states of society.

⁷ Gehrke and Kurz (2006), Kurz (2006) and Kurz and Salvadori (2010) also develop the importance of the “value theory of labour” in Sraffa, although they date the coinage of this term to the mid to late 1940s whereas we are of the opinion that the particular *term* (VTL) was coined in 1955. Irrespective of when the term was introduced, Sraffa certainly worked with unitary wage share price systems as far back as the late 1920s when he was developing his equations, and especially operationalized it in the early 1940s.

⁸ Unless otherwise specified, all equations in the present analysis are conceived in terms of given quantities, i.e. they are *not* the conventional inter-industry equations such that inputs are divided by industry output; hence output is not set equal to unity and the question of returns to scale is simply avoided altogether. This is the level of analysis with which Sraffa himself remained content. On the question of given quantities vs. returns to scale in model-building see Kurz and Salvadori (1995, p. 43).

⁹ “[W]hereas the ‘values’ are by definition expressed...in terms of ‘embodied labour,’ the prices of production are expressed...in terms of ‘labour commanded.’ These two types of quantities of labour will indeed coincide in the limiting case of a zero profit rate” Pasinetti (1977, p. 140).

¹⁰ “An ‘ideal’ system of prices, as understood, for example, by the ‘Ricardian socialists’ (who has claimed at the beginning of the 19th century that the whole net product of an economic system ought to go to the workers), might be $\mathbf{pA} + \mathbf{w}\mathbf{a}_n = \mathbf{p}$. This is a linear system of $(n - 1)$ equations. It determines $(n - 2)$ relative prices and a wage rate which absorbs the entire net product per worker of the economic system. This [is] regarded as the ‘maximum’ wage rate...since it corresponds to a profit rate of zero. We may call it the ‘ideal’ wage rate here, or, from a different point of view, the ‘complete’ wage rate” (Pasinetti 1977, p. 122)

¹¹ In January 1955 the tenth volume of the *Works* was sent to Cambridge University Press, and Sraffa having relinquished his duties for the Royal Economic Society (saving the Index which was not completed until 1973) began his last period of constructive activity by holing himself on the Spanish Island of Majorca. We read from Pollit (1988):

“On 3 January 1955 Dobb reported that ‘Piero is just back from interesting voyagings on the other side of the world; Ricardo Vol. X [should] be out about Feb; and he’s now off for a stay in Majorca – hoping to do some work (non-Ricardo) of his own, tho’ not *too* hopeful that he actually will’...The work that Sraffa hoped to do in Majorca, of course, was begin that process of thought and assembly of past thoughts that eventually emerged as *Production of Commodities by Means of Commodities*. That he would have felt able to do this before the publication of his edition of Ricardo seems unthinkable” (Pollit, 1988, p. 64).

The fruit of Sraffa’s stay on Majorca is a fascinating 31 page handwritten manuscript entitled The Majorca Draft, catalogued as D3/12/52. The Draft constitutes an advanced working draft of Part I of Sraffa’s book (Single Product Industries). In this manuscript Sraffa restates and reformulates much of the material that he had been working on in the second period of scientific activity in the early 1940s.

¹² “It is to be noted that in [the open Leontief] system of equations the matrix of technical coefficients, \mathbf{A} , is of order $(n - 1)$, by contrast with the n th order matrix of the closed system, because it excludes the last column (the consumption coefficients) and the last row (the labor coefficients) of the latter matrix” (Pasinetti, 1977, p. 61). It should be pointed out that by “consumption coefficients” Pasinetti means what we have called the net product, or what in the open Leontief system is called “final demand”; “Final demand might very well consist of both consumption and new investment, provided that it is thought of as given in total...the column vector of the $(n - 1)$ physical quantities... \mathbf{Y} ...mak[es] up the final demand (consumption plus investment), which is assumed to be given” (Pasinetti, 1977, pp. 60-61).

¹³ “In...the capitalist mode of production...what takes place is the opposite of [the law of value].. [a] definite quantity of materialised in a commodity commands a greater quantity of living labour than is contained in the commodity itself” (Marx, 1963 pg. 72).

¹⁴ “The real contribution made by Malthus...is that he places the main emphasis on the *unequal* exchange between capital and wage-labour, whereas Ricardo does not actually explain how the exchange of commodities according to the law of value...gives rise to the unequal exchange between capital and living labour, between a definite amount of accumulated labour and a definite amount of immediate labour, and therefore leaves the origin of surplus value obscure...If one considers the *utilisation* of money or commodities as capital – that is, not their value but their capitalist *utilisation* – it is clear that *surplus-value* is nothing but the

surplus of labour (unpaid labour) which is commanded by capital, i.e. which the commodity or money commands over and above the quantity of labour it itself contains. In addition to the quantity of labour [a magnitude of value-as-capital] contains...it buys a surplus of labour which it itself does not embody” (*TSV III*. Pp. 14-15).

¹⁵ Two quotes from the Old Moor suffice to make this point:

“What competition between the various amounts of capital – which are invested in different spheres of production and have different composition – is striving to produce is *capitalistic communism*, namely that the mass of capital belonging to each sphere of production receives an aliquot part of the total surplus value proportionate to the part of the total social capital which it constitutes” (Marx to Engels 30 April 1868 in Marx and Engels 1975, pg. 193; emphasis in text).

and,

“It is a matter of indifference to the capitalist whether his commodity contains more or less unpaid labour than other commodities, if into its price enters as much of the general stock of unpaid labour, or the surplus product in which it is fixed, as every other equal quantity of capital will draw from the common stock. In this respect *the capitalists are ‘communists’* (Marx 1971, *Theories of Surplus Value, Part III*, pg. 83; emphasis in text).

¹⁶ On the value-side, the maximum rate of profit, here denoted R^p , actually expresses the ratio of two separate “homogenous quantities”, the “quantity of direct to indirect labour employed”, and the “value-ratio of net product to means of production”. In each ratio the numerator and denominator are expressed in quanta of the same units; the former is expressed in quantities of homogenous units of labour, with the numerator denoting the direct living labour added and the denominator indirect embodied labour in the means of production; hence labour-values serve as the *numeraire* for this ratio. The latter “value ratio” is expressed in homogenous quantities of a *numeraire* other than labour. Sraffa notes that in a non-Standard actual system, only when the wage share is unity do the corresponding ratios coincide: “In general (i.e. for all industries that do *not* use the ‘balancing’ proportion) these two ratios will coincide only when the value-ratio is calculated at values for $w = 1$ ” (Sraffa, 1960, p. 17, n. 1). Once the system is “chipped away” so as to be expressed in its Standard proportions, does the physicalist Standard ratio, here denoted R^q , come to coincide with the value ratio. Note that Eatwell (1975) was one of the first to explicitly distinguish the value ratio from the physical ratio, denoting the former R and the latter Q.

¹⁷ “The capitalists pay the workers only the subsistence wage...which represents a fraction $\delta < 1$ of the ‘complete’ wage...What remains, i.e. the fraction $(1 - \delta)$ of [the complete wage], represents the ‘unpaid wage’, or ‘surplus value’ which is appropriated by the capitalists...It may now be noted that the ratio $(1 - \delta)/\delta$...represents what Marx calls the ‘rate of surplus value’, i.e. the unpaid part of [the complete wage] expressed as a percentage fraction of the part which is paid. Or, as it is also said (all the quantities involved being expressed in terms of physical quantities of embodied labor), [the ratio $(1 - \delta)/\delta$] represents the ratio of ‘surplus labor’ to ‘necessary labor.’ Or again, $(1 - \delta)$ also represents the fraction of the working day...in which the worker works for the capitalist, and δ represents the fraction in which he works for himself. This was Marx’s reason for calling their ratio...not only the ‘rate of surplus value’ but also the ‘rate of exploitation’” (Pasinetti, 1977, p. 124).

¹⁸ “For any technology...there exists a standard commodity...which, when standard net product is used as numeraire, yields a relationship between wages and profits identical to that found independently of prices. This is consistent with the classical view that the determination of the distribution of income between wages and profits is logically *prior to*, and independent of prices. Furthermore, it reveals the origin of surplus, in a manner freed from the ambiguities engendered by price calculations” (Eatwell, 1975 p. 548).

¹⁹ Pasinetti accomplishes the very same feat. In section 9 of his appendix on the transformation, Pasinetti considers what happens when the actual (“incomplete”) wage rate is adopted as the *numeraire* vs. that when the “complete” wage rate is adopted as the *numeraire*. The former is given by Equation (V.A.57) on page 140 and the latter by Equation (V.A.58) on page 141. Retaining the inter-industry coefficient framework of Pasinetti (i.e. gross output normalized to unity) and using our notation, these two equations are given by:

$$\mathbf{p} = \mathbf{v}[\mathbf{I} - r\mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}]^{-1}(1 + r) \quad (\text{V.A.57})$$

$$\mathbf{p} = \mathbf{v}[\mathbf{I} - R^{(A+)}\mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}]^{-1} \frac{(1 + R^{(A+)})}{1 + e} \quad (\text{V.A.58})$$

where e = rate of exploitation = unpaid wage ÷ paid wage

Pasinetti remarks on these two equation systems:

“[T]he ‘prices of production’ in (V.A.57) will *always* be greater than the corresponding ‘values,’ excepting in the limiting case in which $r = 0$ and thus $\mathbf{p} = \mathbf{v}$. The reason for this is, whereas the ‘values’ are by definition expressed in terms of ‘embodied labor,’ the prices of production are expressed here in terms of ‘labor commanded.’ These two types of quantities of labor will indeed coincide in the limiting case of a zero profit rate. But, as soon as the rate of profit is positive, each commodity can purchase, in the economic system, a quantity of labor (‘labor commanded’) greater than the quantity of labor embodied in it...By contrast with the prices (V.A.57), the prices (V.A.58) are no longer unilaterally greater than the corresponding ‘values.’ Some prices will prove to be greater than, while others will prove to be smaller than, the corresponding ‘values,’ the borderline case being provided by the composite commodity...which has both a price and a value of unity” (Pasinetti, 1977, pp. 140-142).

The “composite commodity” that Pasinetti references is none other than the fundamental economic normalization of the above analysis, seen by setting the value of the net product equal to its price.

²⁰ We may perhaps understand why Sraffa may have chosen to cross out this diagram, and hence why he chose not to introduce the “pool of profits” in his published work. The answer may reside in Chapter III of his book. There he acknowledges that for an actual economic system, the anarchy of price movements are such that there can be reversals in the direction that we may expect based on the first-order conditions of the LMP ratio. The above unidirectional deviations that come across in both of Sraffa’s diagrams, as well as our own analysis, do so only after the “price-effects” have been rendered moot.

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