Sraffa, Malthus, and the Measure of Value[1]

1. Introduction

If, to quote Stigler, "Ricardo was a fortunate man," then Malthus, to quote Bonar, was "the best abused man of the age". Stigler's characterization of Ricardo notes the latter's fortune as having "lived in a period – then drawing to a close – when an untutored genius could still make economic science" (Stigler, 1953 [1965], p. 302); Bonar's opening caricature of Malthus portrays "a man who defended small-pox, slavery, and child-murder; who denounced soup-kitchens, early marriage, and parish allowances; who 'had the impudence to marry after preaching against the evils of a family'; who thought the world so badly governed that the best actions do the most harm; who, in short, took all romance out of life and preached a dull sermon on the threadbare text – 'Vanity of all vanities, all is vanity" (Bonar 1924, p. 1). Ricardo further had the fortune, notes Stigler, to have "been befriended by Sraffa – who has been befriended by Dobb" (Id). Malthus however suffered the further misfortune of never having garnered such a friendship with Mr. Sraffa; or did he?

This essay is admittedly one of conjecture, albeit informed, and a heretical conjecture at that. Heretical that is as regards the standard story of Sraffa's analysis as solely and exclusively steeped in Ricardo, one that must necessarily be diametrically-opposed to any hint of Malthus. Yet for all the Ricardian trappings identified with Sraffa, and despite all the "neo-Ricardian" elements of analysis identified with the "Sraffian" followers of Sraffa, there remains the fact that, in the end, when it all boiled down, Sraffa "surprising[ly]" identified the Standard commodity with that very un-Ricardo-like measure of value, "namely labour commanded" (Sraffa 1960, p. 94, Appendix D).

Bonar's portrayal of Malthus of course is by his own admission a caricature.[2] Indeed Bonar's intellectual biography of the Right Honorable Parson is just one in a line of such expositions[3] on the life and work of Thomas Robert Malthus that speaks not only to the latter's genteelness, kindness,

and warmness, but also to the rigor and carefulness of the manner of inquiry and continuous critical self-reflection that Malthus constantly adhered to, especially in the face of empirical evidence.[4] Even Marx, whom Schumpeter remarks "poured on [Malthus] vitriolic wrath" (1954, p. 480), recognizes this aspect of Malthus's work.[5]

Hence we find much merit in revisiting Malthus's question on the measure of value. The remainder of this essay is as follows. In the next section the question of whether Sraffa meant Smith or Malthus regarding the labour commanded concept referred to in Appendix D of *Production of Commodities* is explored, where archival evidence reveals that it was the latter, not the former, that "commanded" the attention of Sraffa. The following section attempts to demonstrate that for Sraffa, the labour commanded concept indeed refers to the final notion of the measure of value that Malthus settled upon, namely that of the "constant labour that produces the wages and profits in a commodity". There Malthus's original table taken from his 1823 work *The Measure of Value Stated and Illustrated* is extended and a diagrammatic depiction is advanced. Malthus's table as it appears in 1823 is reproduced in Appendix A for convenience of the reader, and in Appendix B the table is subjected to further extension. The final section concludes.

2. Labour Commanded in Sraffa's Appendix D: Smith or Malthus?

The question of labour commanded in Sraffa is one of the more perplexing and least developed lines of inquiry. In general the notion of labour commanded has long been the subject of wide interpretation and disagreement, unlike the more definitive notion of labour embodied upon which everybody agrees.[6] In his published work he wrote very little on the subject, and in this writer's opinion leaves us all with a cryptic parting shot in Appendix D: References to the Literature. There he remarks that the development of the three invariable Standards of value in his book leads, he notes with irony, to the labour commanded Standard:

> "The concept of a standard measure of value as a medium between two extremes (§ 17 ff.) also belongs to Ricardo and it is surprising that the Standard commodity which has been evolved from it here should be found to be equivalent to something very close to the standard suggested by Adam Smith, namely 'labour commanded' (§43), to which Ricardo himself was so decidedly opposed" (Sraffa 1960, p. 94).

Yet we are confronted with the change in the manner of presentation vis-à-vis his unpublished notes vs. the final form his argument takes in his book. In drafts of the Historical Appendix written in 1959 (archived as D3/12/98), Sraffa identifies *not* Adam Smith but rather Ricardo's "opponents" as those who advanced the 'labour commanded' concept closely related to the Standard commodity :

> "The concept of a standard measure of value as a medium between two extremes (§ 17 ff.) belongs to Ricardo and it is surprising that the standard commodity which has evolved from it here should be found to be equivalent to something very close to the standard suggested *by his opponents*, namely 'labour commanded' (§ 43)" (D3/12/98/1; emphasis added).

We find inserted in another draft of this passage the following disclaimer on the definition of "labour commanded"

> "[A]lthough this phrase which has been given slightly different meaning by its proponents is here defined in yet another way…although the variety of senses in which this phrase has been used…although this variously ambiguous phrase must receive yet another definition" (D3/12/98/15).

Clearly by "Ricardo's opponents" Sraffa here is referring in the main to Malthus, and perhaps to a lesser extent followers of Malthus such as John Cazenove. We think that the change from Ricardo's "opponents" to Adam Smith in the published account is not a trivial point. Certainly the command

concept originates with Smith, and Malthus definitely embraces it with vigor. But Malthus's particular approach to the command concept is not the same as Smith's, and represents in certain important respects a significant advance over that of our 18th century Scottish Moral Philosopher. This is especially true after the first edition of his *Principles of Political Economy* (1820)[7], in *The Measure of Value Stated and Illustrated* of 1823, *Definitions in Political Economy* of 1827, and in two lesser known lectures given in fulfillment of his duties as an associate of the Royal Society of Literature; the first read in May 1825 entitled "On the Measure of the Conditions necessary to the Supply of Commodities" and the second read in November 1827 entitled "On the meaning which is most usually and most correctly attached to the term "Value of a Commodity."[8]

Malthus's opinions on the labour commanded standard constantly changed and developed over time. Indeed, we find in the Index to Ricardo's Works (*Works XI*) that Sraffa attributed to Malthus *five distinct conceptions of the measure of value*:

a. Labour commanded
b. Pay of a day's labour
c. Mean between corn and labour
d. Silver (or gold) picked up in a day on the sea-shore
e. The constant labour that produces the wages and profits in a commodity

(Sraffa, *Works XI*, p. 107).

We are of the opinion that the last measure, "the constant labour that produces the wages and profits in a commodity", is very much is in the spirit of the definition of labour commanded that Sraffa attributes to the Standard commodity.

3. Labour commanded by both wages and profits

Hence an important aspect of Malthus's approach to labour commanded is the idea, not found in Smith, that labour command applies equally well to the profit relation as to the wage relation.[9] Here the importance of the profit share become explicit, something that Joan Robinson understood in her development of the theory of exploitation:

> "We have a theory of the share of profit – the rate of exploitation. The *share* of profit is far more important than the *rate* of profit. The *share* of profit is something that actually happens and affects people's lives - the rate of profit is a mental calculation" (Robinson, 1980b, pg. 186).[10]

Thus wage revenue will command a certain quantum of living labour, *and so too will profit revenue*. This Malthus explicitly says in his 1825 lecture:

> "However various may be the ingredients of the capital employed, or for however long or short a time they may be advanced, if we exclude rent and taxes, the conditions of supply of commodities consists exclusively of the labour and profits necessary to their production, and these two together are exactly represented by the quantity of labour which *they* will ordinarily and on average command" (Malthus, 1829, p. 177; emphasis added).

Notice the plural "*they*…will command"; where "they" clearly refers to *both* wages *and* profits; and of course by definition the quantum of labour a particular level of wage revenue (wages at a regime of functional income distribution) will command will be exactly the same as the quantum of labour the associated level of profit revenue (profits at the same level of function income distribution) commands as well. And both will be equal to the quantum of living labour, L, bestowed. Ricardo in a letter to Malthus in April, 1823 would call this the "constant constant labour that produces the

wages and profits in a commodity" (*Works IX*, pg. 280), which as noted is the final measure of value that Malthus settled upon, writing to Ricardo in July 1823 of its "intuitive proof" (*Works IX* p. 308).[11]

Malthus attempted to illustrate this proposition in his 1823 *Measure of Value Stated and Illustrated*, Ricardo's notes on which was published as Porta (1992)[12]. Although his exposition is generally to be regarded as a failure,[13] that he was broaching on the remunerative-extractive relationship can be shown in a simple extension of Malthus's numeric example that appears on the page 38 of Malthus's 1823 book.[14]

Recent inspection of Malthus's table has raised some interesting questions and notions. Glyn (2006) for example observes that:

> "Despite Malthus' quite unconvincing exposition, it is remarkable that the table actually calculates wages and profits in terms of the amount of labour required to produce what workers and capitalists receive…This is exactly how Ricardo represents wages and profits in the *Principles*…Moreover, Malthus' 'labour-value' calculations are derived in the table from the physical conditions of production and the real-wage, a step which neither Ricardo nor Marx explicitly carried out. The one-sector corn model permits this to be done without simultaneous calculations of labour values in several sectors, and Malthus works out the division of the product of in labour value terms for a range of assumptions about wage and productivity levels. *Thus, we have in Malthus's table, explicit calculations of what Marx would later describe as surplus value, measured in labour time, and derived from the conditions of production and the real wage*" (Glyn 2006, p. 311; emphasis added)

We are in agreement in the main with Glyn's sentiments, and to the following discussion the reader is turned to the Appendices of the present paper: Appendix A contains a replication of the original

table in Malthus [1823], as reproduced from Porta (1992, p. 34). Appendix B contains an extension that we make on Malthus's original table. There it is demonstrated that Glyn's remark of "explicit calculations of what Marx would…describe as surplus value" is borne out by inspection of Columns (X : 5) consisting of *paid labour time* and (XI : 6)[15] consisting of *unpaid labour time*.

The underlying model Malthus presents here is in truth *not* a corn model in the sense that there is no seed input required to produce the output. "Corn" here is thus the product of "unassisted labour", something that Malthus attributed in correspondence with Ricardo to "gold picked up at the seashore". Hence the price equation implicit in Malthus's table can be expressed as[16]:

$$p_{ij} = (1+r)\, w_{ij} L$$

where: p_{ij} = corn-price of output produced by capital i on land type j

w_{ij} =capital i employed on land type j

r = rate of profit = rate of exploitation = e

Because this is a pure wage determined model, capital will consist exclusively of wages advanced and the rate of profit (r) will be the rate of exploitation (e) and hence will be upper-bounded by infinity. This will be in marked contrast with models that include capital-inputs proper such that the maximum profit rate is a determined value, and here the importance of the ever-present commodity residue becomes explicit.

We also concur with Glyn in that albeit a cumbersome and perhaps crude illustration, it nonetheless represents an attempt at rigor and demonstration of the robustness of Malthus's propositions. Malthus explicitly varies *both* the fertility of land *and* wages on each plot, and hence engages in a rudimentary inter-(across land types) and intra-(within land types) disequilibrium analysis, and

"shows" (actually he simply assumes) that despite these varied disequilibria the same quantum of value added is that of the labour of ten men.

In order to uncover certain analytical properties of Malthus's illustration, it is useful to bite off far less than Malthus did by considering not the question "invariable labour" across the varied conditions of production (e.g. land and wage-as-capital), but rather what happens at a single level of productivity and land fertility (or *technique*) and varying the wage share, retaining the conceptual structure of our extension of Malthus's table. Here wages are determined according to the value of the wage share. This we do in the following table:

[Insert Table 1]

When the underlying model of Malthus's table is explicit, we see precisely that Glyn's contention concerning the aggregate of wage and profits equaling productivity evidenced in the sum of columns (6) and (7) at every distributive share. Turning to the quantity of labour commanded by each revenue form, we see from columns (8) and (9) that there is a inverse perfect symmetry between labour commanded by wages (LC_w) when compared to labour commanded by profits (LC_r): when the wage share is unity, the quantity of labour commanded by the wage is exactly equal to the quantum of labour bestowed (10 L), whereas labour commanded by profit revenue is infinite (since it is associated with a zero profit); and when the wage share is zero the roles are reversed and the labour commanded at the zero wage is infinite and the labour commanded by profit revenue is equal to the quantum of labour bestowed (10 L).

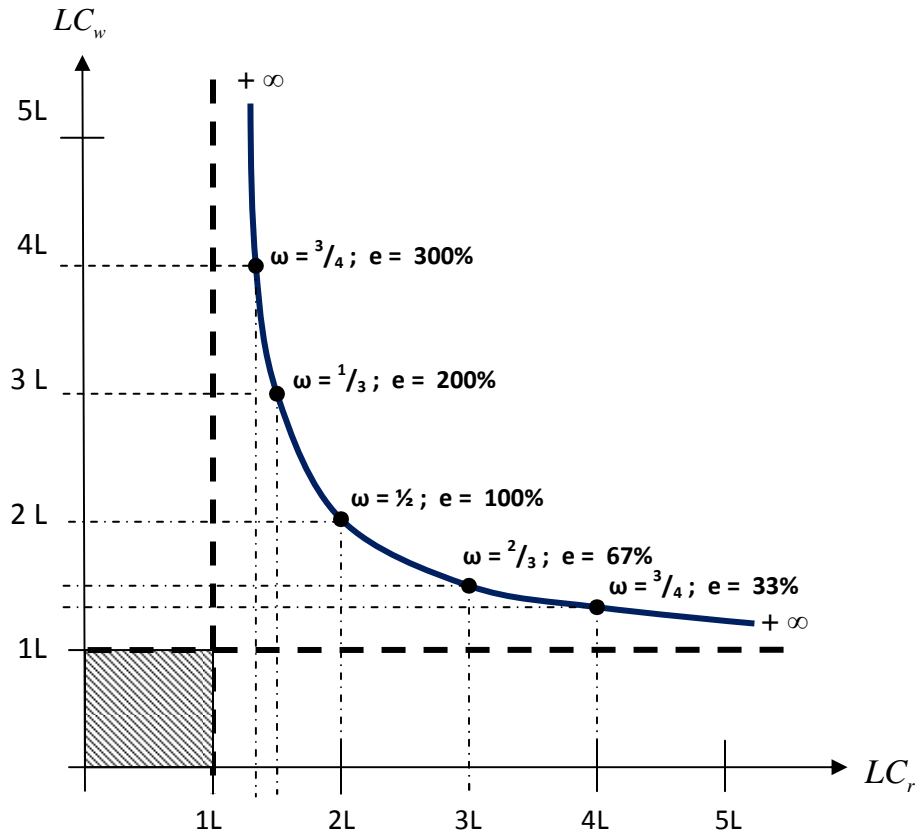
Hence we find that at every level of the wage share, there is a unique and perfectly symmetric inverse relation between the respective quantities of labour commanded by the revenues of each distributive share: the labour that produces wages and profits *is constant*. Each of these points of symmetry will be a unique function of the rate of exploitation, which here is also the rate of profit

due to the assumption of "unassisted labour". In a letter to Ricardo dated 21 July, 1823, Malthus refers to this as an "intuitive proof" that labour is invariable:

> "The proof which may be said almost to be of the class called intuitive is in reality complete without the table or the parts immediately connected with it. These parts merely supply a further confirmation of the truth by answering the following question. How comes it that labour should remain of the same value in the progress of society, when it is known that it must require more labour to produce it? And the answer to this question is, that as profits depend upon the *proportion* of the whole produce which goes to labour, it must necessarily happen that the increase of value occasioned by the additional quantity of labour will be exactly counterbalanced by the diminution in the amount of profits, leaving the value of labour the same" (*Works IX,* pp. 308-9).

We can generalize this tradeoff between the respective quantities of labour commanded by setting the total quantity of (invariable) labour equal to one unit (1 L), an inverse relation shown in Figure 1 below:

Figure 1: Labour commanded by factor revenues (labour commanded-factor revenue space)

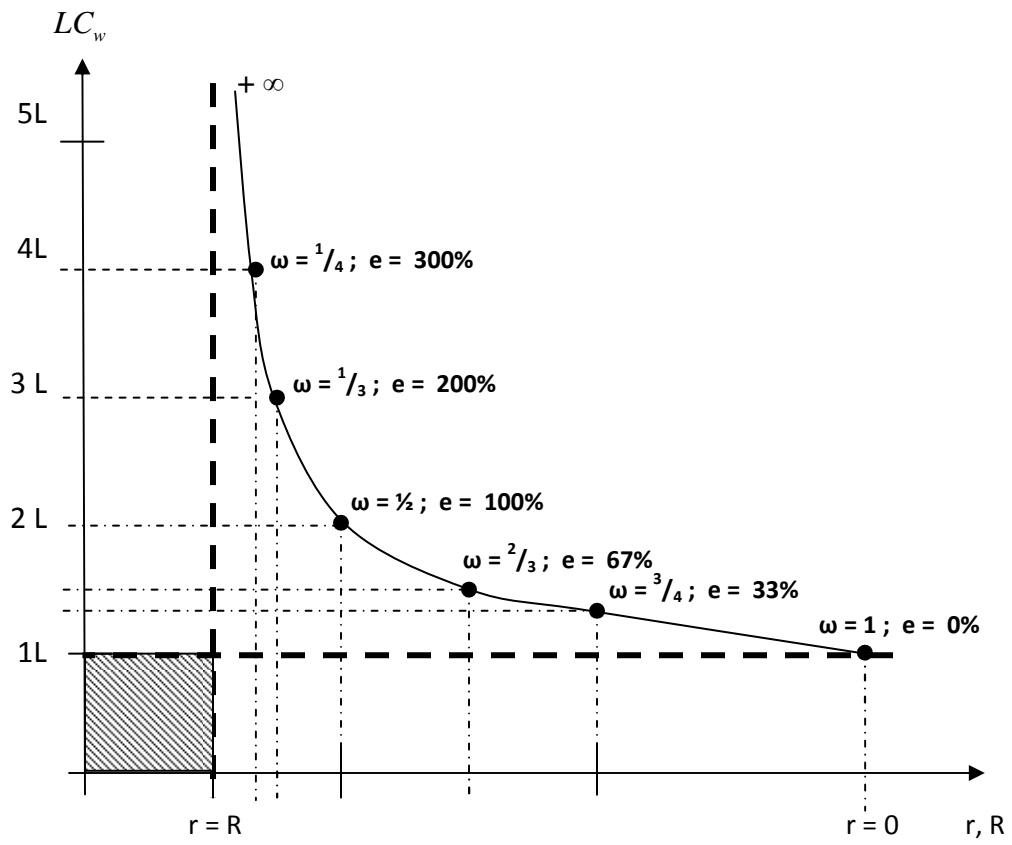


The symmetric tradeoff between the quantities of labour that the respective distributive revenues will command is thus depicted in the above rectangular hyperbola asymptotically approaching (on *both* sides of the revenue stream) the "invariable" quantum of living labour added, here normalized to one unit of L. This fundamental inverse relationship simply illustrates the inverse wage-profit relation.

However once we move into systems of production where living labour is "assisted" by means of production requirements, the rate of profit no longer corresponds to the rate of exploitation as the profit mass will be assessed according to the value of the means of production. It is here that we find the maximum rate of profit concept ($r^{\omega=0} = r^{r=R} = R$), a fundamental analytical point for Sraffa. Thus in Figure 2 the horizontal axis represents the value of the rate of profit from its minimum

value of zero, where the labour commanded by wages is by definition equal to the living labour, to its maximum value of R, where the labour commanded by wages approaches infinity.

Figure 2: Labour commanded by wage revenue at different levels of the rate of profits



Hence Sraffa moved away from the idea that labour commanded is conceived only with respect to the wage relation, and seems to argue that identification of the labour commanded standard with the wage relation only reflects a mimicking of the quest for an invariable measure for Ricardo's bestowed standard. This we find articulated in a very cryptic footnote in his Introduction to *Works I* that references Ricardo's procedure of "identifying the problem of a measure of value with that of the Law of value" (Sraffa, 1951, pg. *xli*):

> "This idea that to every theory there corresponds an appropriate 'invariable measure' is evidently based on Ricardo's experience with his own theory, where to the determination of

> value by embodied labour there corresponds an invariable measure in the shape of a commodity produced by a constant quantity of labour; and in so far as there are exceptions to the theory, to the same extent the accuracy of the measure is affected. This correspondence, however, is a *peculiar* property of Ricardo's theory and does not necessarily apply to other theories. Thus there would not seem to be such a relation between the theory that wages determine prices and the 'labour commanded' standard" (Sraffa, 1951, pg. *xli*; emphasis added).

Notice the emphasis on the word "peculiar", because that is precisely the same term he uses to characterize his own unique approach to labour commanded, although he never clearly spells out exactly what that approach is.

4. Conclusion

It is our conjecture that Sraffa seems to argue that the labour commanded concept need not necessarily be construed on the wage-side of the ledger exclusively. Rather Sraffa conceives of command in terms of the quantities of labour of all aspects of the price form, both its summed totality and its composite parts. Hence wages will command a quantity of labour; profits too will command a quantity of labour; so will the net product; even the means of production themselves, now expressed as capital, commands quantities of labour, albeit *unpaid labour*.[17] The command concept, in other words, can be seen as representing a more general phenomenon in commodity-producing societies. And this is where we argue that Sraffa's more fundamental critique belongs: he shows that there is a fundamental symmetry between the quantity of labour bestowed in the value-added by living labour and the quantity of labour commanded by the resulting net product this productive living labour creates, no matter the division of the product between wages and profits. We have also attempted to show that this is in the spirit of and indeed can be construed as an

extension of the final measure of value that Malthus settled upon, namely the "constant labour that produces the wages and profits in a commodity".

Appendix A: Table A: Malthus's [1823] Table illustrating the invariable Value of Labour and its Results (exact replication of original)[18]

[Insert Table A]

Appendix B: An extension of Malthus's [1823] Table on Invariable Labour in *The Measure of Value of Value Stated and Illustrated*

The following represents an extension of Mathus's Table in his 1823 *Measure of Value Stated and Illustrated*. Our extended table consists of Roman numeral columns of 13 different categories, 8 of which appear in Malthus's original table, for 14 different capitals (here defined strictly in terms of wages). The eight common categories with Malthus are indicated by the arithmetic numbers after the appropriate Roman numerals. Thus (II : 8) indicates that our column II is Malthus's column 8. Whenever one of Malthus's categories is presented the exact definition as appears in his table is listed, along with our own definition and/or variable specification.

Malthus's numeric example conceives prices are purely wage-determined, according to the equation $p_j = (1+r)\,wL_j$, where j = the quality of land that the homogenous output "corn" is produced. Because it is a purely living-labour determined price structure, the rate of profit will by definition also be the rate of exploitation and hence bounded by zero and positive infinity.

[Insert Table B1]

It should be noted that strictly speaking this is *not* a corn-model in the sense of Ricardo's due precisely to the fact that Malthus's model is a purely living labour model - hence there is no seed - and for Ricardo the seed input was explicit. It would much more appropriate to think of this model as an example of what Malthus would term the "product of unassisted labour"; an example of such a commodity Malthus used in correspondence with Ricardo was "gold picked up by the seashore".

The breakdown of this extended table is as follows. Row (I) lists the land types of different fertility given in Malthus's example.[19] There exist 7 different land fertilities A through G, all of which employ ten men expressed in units L, as seen in our Column (II) (Malthus's Column 8). The output of each land type is given in our Column (III) (Malthus's Column 1), expressed in quarters of "corn".

In our Column (IV) we express the productivity of labour of each land type from the data provided in Malthus's original table. Column (V) (Malthus's column 2) shows the given annual wages per worker for each land type. Notice here that the most fertile land A employs three different capitals[20], lands B through F each employ two different capitals, and the marginal land G employs only one capital. The "corn" wage is given, for Malthus by conditions of "demand and supply", and is expressed in the same units as labour's productivity ($^{qtr}/_{L}$). We dub this the "corn relative wage" because it represents the proportionate wage in Ricardo and accordingly is the portion of the productivity of labour that is remunerated to the labourers. From here we easily calculate the implicit wage share in this example, shown in Column (VI). This wage share will figure very prominently in the remainder of the table. Column (VII) (Malthus's column 3) gives the wage bill for each capital and column (VIII) gives the productivity wage ratio, an important ratio for the command of both wages and profits. In our Column (IX) the rate of profit is calculated, here in terms of the ratio of distributed income shares. As indicated, since Malthus's example is void of means of production requirements (i.e. unassisted labour), the rate of profit conflates to the rate of exploitation. Columns (X) and (XI) (Malthus's 5 and 6) represents the quantity of labour necessary to produce the wage and profit revenues of each does of capital. In our model it is ascertained by multiplying the implicit wage share against the constant quantity of labour. As seen the sum of the two is equal to the constant quantity of labour = 10 L. For Malthus this was *the* fundamental result – that irrespective the various conditions and the various divisions of wages and profits, the sum of the labour necessary to produce each of latter two income categories will equal the total (constant) quantity of labour employed. Finally columns (XII) (Malthus's penultimate column 9) and Column XIII (which does not appear in Malthus) Both quantities of labour commanded will, under capitalistic regimes of distribution, will command a greater quantum of labour than the total L employed. Below is our extension of Malthus's original table. Nothing has been added to the original example; only ratios and products have been constructed based on the original data. One difference in this extended

version is the explicit identification of the appropriate units of each category (L, qtr, $^{qtr}/_{L}$); absent any units indicates scalar value (i.e. ω, $^{1}/_{\omega}$, r, and $^{1}/_{r}$).

[Insert Table B2]

Table 1: Malthus's Measure of Value illustration modified across regimes of distribution for single technique

(1) Quantity of living labour (L)	(2) Net Product (Y_{np_A})	(3) Productivity of labour $\left(y_A = \frac{qtr}{L}\right)$	(4) Wage share (ω_A)	(5) Rate of Profit = rate of exploitation $\left[r = e = \frac{(1-\omega)}{\omega}\right]$	(6) Corn-real wage (paid labour) $\left[w_A = \omega y = \frac{qtr}{L}\right]$	(7) Corn-real profits (unpaid labour) $\left[z_A = (1-\omega)\, y = \frac{qtr}{L}\right]$
10 L	150 qtr	$^{150\ \text{qtr}}/_{10\ \text{L}}$ $= 15\ ^{\text{qtr}}/_{\text{L}}$	$\omega^{r=0} = \omega^0 = 1$	$r^{r=0} = e^0 = 0$	$w^{r=0} = w^0 = 15\,^{qtr}/_{L}$	$z^{r=0} = z^0 = 0\,^{qtr}/_{L}$
			$\omega^{3/4} = \frac{3}{4}$	$r^{3/4} = 33.34\%$	$w^{3/4} = 11.25\,^{qtr}/_{L}$	$z^{3/4} = 3.75\,^{qtr}/_{L}$
			$\omega^{2/3} = \frac{2}{3}$	$r^{2/3} = 50\%$	$w^{2/3} = 10\,^{qtr}/_{L}$	$z^{2/3} = 5\,^{qtr}/_{L}$
			$\omega^{1/2} = \frac{1}{2}$	$r^{1/2} = 100\%$	$w^{1/2} = 7.5\,^{qtr}/_{L}$	$z^{1/2} = 7.5\,^{qtr}/_{L}$
			$\omega^{1/3} = \frac{1}{3}$	$r^{1/3} = 200\%$	$w^{1/3} = 5\,^{qtr}/_{L}$	$z^{1/3} = 10\,^{qtr}/_{L}$
			$\omega^{1/4} = \frac{1}{4}$	$r^{1/4} = 300\%$	$w^{1/4} = 3\,^{qtr}/_{L}$	$z^{1/4} = 11.25\,^{qtr}/_{L}$
			$\vdots$	$\vdots$	$\vdots$	$\vdots$
			$\omega^{1/100} = \frac{1}{100}$	$r^{1/100} = 9900\%$	$w^{1/100} = .15\,^{qtr}/_{L}$	$z^{1/100} = 14.85\,^{qtr}/_{L}$
			$\vdots$	$\vdots$	$\vdots$	$\vdots$
			$\omega^{r=R} = 0$	$r^{r=R} = \infty$	$w^{r=R} = 0\,^{qtr}/_{L}$	$z^{r=R} = 15\,^{qtr}/_{L}$

$\sum = y = w + z = 15\frac{qtr}{L}$

(invariable labour productivity)

(4) Wage share (ω_A)	(8) Labour commanded by wage revenue $\left[LC_r = \left(\frac{y}{z}\right)(10L)\right]$	(9) Labour commanded by profit revenue $\left[LC_r = \left(\frac{y}{z}\right)(10L)\right]$
$\omega^{r=0} = \omega^0 = 1$	$(LC_w)^0 = \left(\frac{15\ \text{qtr/L}}{15\ \text{qtr/L}}\right)(10L) = 10\,L$	$(LC_r)^0 = \left(\frac{15\ \text{qtr/L}}{0\ \text{qtr/L}}\right)(10L) = \infty\,L$
$\omega^{3/4} = \frac{3}{4}$	$(LC_w)^{3/4} = \left(\frac{15\ \text{qtr/L}}{11.25\ \text{qtr/L}}\right)(10L) = 13.34\,L$	$(LC_r)^{3/4} = \left(\frac{15\ \text{qtr/L}}{3.75\ \text{qtr/L}}\right)(10L) = 40\,L$
$\omega^{2/3} = \frac{2}{3}$	$(LC_w)^{2/3} = \left(\frac{15\ \text{qtr/L}}{10\ \text{qtr/L}}\right)(10L) = 15\,L$	$(LC_r)^{2/3} = \left(\frac{15\ \text{qtr/L}}{5\ \text{qtr/L}}\right)(10L) = 30\,L$
$\omega^{1/2} = \frac{1}{2}$	$(LC_w)^{1/2} = \left(\frac{15\ \text{qtr/L}}{7.5\ \text{qtr/L}}\right)(10L) = 20L$	$(LC_w)^{1/2} = \left(\frac{15\ \text{qtr/L}}{7.5\ \text{qtr/L}}\right)(10L) = 20L$
$\omega^{1/3} = \frac{1}{3}$	$(LC_w)^{1/3} = \left(\frac{15\ \text{qtr/L}}{5\ \text{qtr/L}}\right)(10L) = 30\,L$	$(LC_w)^{1/3} = \left(\frac{15\ \text{qtr/L}}{10\ \text{qtr/L}}\right)(10L) = 15\,L$
$\omega^{1/4} = \frac{1}{4}$	$(LC_w)^{1/4} = \left(\frac{15\ \text{qtr/L}}{3.75\ \text{qtr/L}}\right)(10L) = 40\,L$	$(LC_w)^{1/4} = \left(\frac{15\ \text{qtr/L}}{11.25\ \text{qtr/L}}\right)(10L) = 13.34\,L$
$\vdots$	$\vdots$	$\vdots$
$\omega^{1/100} = \frac{1}{100}$	$(LC_w)^{1/100} = \left(\frac{15\ \text{qtr/L}}{.15\ \text{qtr/L}}\right)(10L) = 1000\,L$	$(LC_w)^{1/100} = \left(\frac{15\ \text{qtr/L}}{14.85\ \text{qtr/L}}\right)(10L) = 10.10\,L$
$\vdots$	$\vdots$	$\vdots$
$\omega^{r=R} = 0$	$(LC_w)^{r=R} = \left(\frac{15\ \text{qtr/L}}{0\ \text{qtr/L}}\right)(10L) = \infty\,L$	$(LC_w)^{r=R} = \left(\frac{15\ \text{qtr/L}}{15\ \text{qtr/L}}\right)(10L) = 10\,L$

Table A: Malthus's [1823] Table illustrating the invariable Value of Labour and its Results (exact replication of original)

1.	2.	3.	4.	5.	6.	7.	8.	9.
Quarters of corn produced by Ten Men on varying fertility of the soil = *Net product* $(Y_{np} =$ qtr of corn$)$	Yearly corn wages to each Labourer determined by demand and supply = *Corn relative wage* $\left(w = {}^{qtr}\!/_{L}\right)$	Advances in Corn Wages, or variable Produce commanding the Labour of Ten Men = *Wage bill (paid labour)* $(W = wL)$	Rate of profits under the foregoing Circumstances *(here = rate of exploitation)* $\left[r = \frac{(1-\omega)}{\omega} = \%\right]$	Quantity of labour required to produce the wages of ten men = *number of labourers necessary to produce wage bill* $\left[L_w = (\omega) L\right]$	Quantity of profits on the advanced of labour = *number of labourers necessary to produce profit mass* $\left[L_r = (1-\omega) L\right]$	Invariable Value of the Wages of a given Number of Men = *Quantity of Labour* = *living labour added* $(L =$ # of workers$)$	Value of 100 Quarters of Corn under the varying Circumstances supposed.	Value of Product of the Labour of Ten Men under the Circumstances Supposed = *Labour commanded by wage revenue* $\left[LC_w = \left(\frac{y}{w}\right)L = \left(\frac{1}{\omega}\right)L\right]$
$\left(Y_{np}\right)_j = qtr$	$w_j^{(i)} = {}^{qtr}\!/_{L}$	$W_j^{(i)} = \left(w_j^{(i)}\right)L = qtr$	$r_j^{(i)} = \left[\frac{(1-\omega)}{\omega}\right]_j^{(i)}$	$(L_w)_j^{(i)} = \left(\omega_j^{(i)}\right)L < L$	$(L_r)_j^{(i)} = \left(1-\omega_j^{(i)}\right)L < L$	$L_j^{(i)} = 10\ L$		$(LC_w)_j^{(i)} = \left(\frac{1}{\omega}\right)_j^{(i)} L > L$
150 *qrs.*	12 *qrs.*	120 *qrs.*	25 pr. Ct.	8	2	10	8.33	12.5
150	13	130	15.38	8.66	1.34	10	7.7	11.53
150	10	100	50	6.6	3.4	10	10	15
140	12	120	16.66	8.6	1.4	10	7.14	11.6
140	11	110	27.2	7.85	2.15	10	9.09	12.7
130	12	120	8.3	9.23	0.77	10	8.33	10.8
130	10	100	30	7.7	2.3	10	10	13
120	11	110	9	9.17	0.83	10	9.09	10.9
120	10	100	20	8.33	1.67	10	10	12
110	10	100	10	9.09	.91	10	10	11
110	9	90	22.2	8.18	1.82	10	11.1	12.2
100	9	90	11.1	9	1	10	11.1	11.1
100	8	80	25	8	2	10	12.5	12.5
90	8	80	12.5	8.88	1.12	10	12.5	11.25

Table B1: Table illustrating the invariable Value of Labour and its Results (extended)

(I) Land Type	**(II : 8)** "Invariable Value of the Wages of a given Number of Men" Quantity of Labour = living labour added ($L =$ # of workers)	**(III : 1)** "Quarters of corn produced by Ten Men on varying fertility of the soil" = Net product ($Y_{np} =$ qtr of corn)	**(IV)** Productivity of labour $\left(y = {Y_{np}}/{L} = {qtr}/{L}\right)$	**(V : 2)** "Yearly corn wages to each Labourer determined by demand and supply" = Corn relative wage $\left(w = {qtr}/{L}\right)$	**(VI)** Implicit wage share (wage per land productivity) $\left(\omega = {w}/{y}\right)$	**(VII : 3)** "Advances in Corn Wages, or variable Produce commanding the Labour of Ten Men" = Wage bill (paid labour) $(W = wL)$	**(VIII)** Productivity -wage ratio $\left({y}/{w}\right) = \left(\frac{1}{\omega}\right)$
j	$L_j^{(i)} = 10\,L$	$\left(Y_{np}\right)_j = qtr$	$y_j = {qtr}/{L}$	$w_j^{(i)} = {qtr}/{L}$	$\omega_j^{(i)} = \frac{w}{y}$	$W_j^{(i)} = \left(w_j^{(i)}\right)L = qtr$	$\left(\frac{1}{\omega}\right)_j^{(i)}$

(A) Land Type	**(IX : 4)** "Rate of profits under the foregoing Circumstances" (here = rate of exploitation) $\left[r = \frac{(1-\omega)}{\omega} = \%\right]$	**(X : 5)** "Quantity of labour required to produce the wages of ten men" = number of labourers necessary to produce wage bill (paid labour-time) $\left[L_w = (\omega)\,L\right]$	**(XI : 6)** "Quantity of profits on the advanced of labour" = number of labourers necessary to produce profit mass (unpaid labour-time) $\left[L_r = (1-\omega)\,L\right]$	**(XII : 9)** "Value of Product of the Labour of Ten Men under the Circumstances Supposed" = Labour commanded by wage revenue $\left[LC_w = \left(\frac{y}{w}\right)L = \left(\frac{1}{\omega}\right)L\right]$	**(XIII)** Labour commanded by profit revenue $\left[LC_r = \left(\frac{y}{w}\right)\left(\frac{1}{r}\right)L = \left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)L\right]$
j	$r_j^{(i)} = \left[\frac{(1-\omega)}{\omega}\right]_j^{(i)}$	$\left(L_w\right)_j^{(i)} = \left(\omega_j^{(i)}\right)L < L$	$\left(L_r\right)_j^{(i)} = \left(1-\omega_j^{(i)}\right)L < L$	$\left(LC_w\right)_j^{(i)} = \left(\frac{1}{\omega}\right)_j^{(i)} L > L$	$\left(LC_r\right)_A^{(i)} = \left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_A^{(i)} L > L$

$$\underbrace{\qquad\qquad\qquad\qquad}_{L_w + L_r = 10L}$$

(invariable labour)

Table B2: Table illustrating the invariable Value of Labour and its Results (extended)

(I) Land Type	**(II : 8)** "Invariable Value of the Wages of a given Number of Men" Quantity of Labour = living labour added ($L =$ # of workers)	**(III : 1)** "Quarters of corn produced by Ten Men on varying fertility of the soil" = Net product ($Y_{np} =$ qtr of corn)	**(IV)** Productivity of labour $\left(y = {}^{Y_{np}}\!/_{L} = {}^{qtr}\!/_{L}\right)$	**(V : 2)** "Yearly corn wages to each Labourer determined by demand and supply" = Corn relative wage $\left(w = {}^{qtr}\!/_{L}\right)$	**(VI)** Implicit wage share (wage per land productivity) $\left(\omega = {}^{w}\!/_{y}\right)$	**(VII : 3)** "Advances in Corn Wages, or variable Produce commanding the Labour of Ten Men" = Wage bill (paid labour) $(W = wL)$	**(VIII)** Productivity -wage ratio $\left({}^{y}\!/_{w}\right) = \left(\frac{1}{\omega}\right)$
A	$L_A^{(i)} = 10\,L$ $L_A^{(ii)} = 10\,L$ $L_A^{(ii)} = 10\,L$	$\left(Y_{np}\right)_A = 150\ qtr$	$y_A = {}^{150qtr}\!/_{10L} = 15\,{}^{qtr}\!/_{L}$	$w_A^{(i)} = 12\,{}^{qtr}\!/_{L}$ $w_A^{(ii)} = 13\,{}^{qtr}\!/_{L}$ $w_A^{(iii)} = 10\,{}^{qtr}\!/_{L}$	$\omega_A^{(i)} = \frac{\left(12\,{}^{qtr}\!/_{L}\right)}{\left(15\,{}^{qtr}\!/_{L}\right)} = .8$ $\omega_A^{(ii)} = \frac{\left(13\,{}^{qtr}\!/_{L}\right)}{\left(15\,{}^{qtr}\!/_{L}\right)} = .867$ $\omega_A^{(iii)} = \frac{\left(10\,{}^{qtr}\!/_{L}\right)}{\left(15\,{}^{qtr}\!/_{L}\right)} = .667$	$W_A^{(i)} = \left(w_A^{(i)}\right)L = 120\ qtr$ $W_A^{(ii)} = \left(w_A^{(ii)}\right)L = 130\ qtr$ $W_A^{(i)} = \left(w_A^{(iii)}\right)L = 100\ qtr$	$\left(\frac{1}{\omega}\right)_A^{(i)} = 1.25$ $\left(\frac{1}{\omega}\right)_A^{(ii)} = 1.15$ $\left(\frac{1}{\omega}\right)_A^{(iii)} = 1.5$
B	$L_B^{(i)} = 10\,L$ $L_B^{(ii)} = 10\,L$	$\left(Y_{np}\right)_A = 140\ qtr$	$y_B = {}^{140qtr}\!/_{10L} = 14\,{}^{qtr}\!/_{L}$	$w_B^{(i)} = 12\,{}^{qtr}\!/_{L}$ $w_B^{(ii)} = 11\,{}^{qtr}\!/_{L}$	$\omega_B^{(i)} = \frac{\left(12\,{}^{qtr}\!/_{L}\right)}{\left(14\,{}^{qtr}\!/_{L}\right)} = .857$ $\omega_B^{(ii)} = \frac{\left(11\,{}^{qtr}\!/_{L}\right)}{\left(14\,{}^{qtr}\!/_{L}\right)} = .786$	$W_B^{(i)} = \left(w_B^{(i)}\right)L = 120\ qtr$ $W_B^{(ii)} = \left(w_B^{(ii)}\right)L = 110\ qtr$	$\left(\frac{1}{\omega}\right)_B^{(i)} = 1.16$ $\left(\frac{1}{\omega}\right)_B^{(ii)} = 1.27$
C	$L_C^{(i)} = 10\,L$ $L_C^{(ii)} = 10\,L$	$\left(Y_{np}\right)_A = 130\ qtr$	$y_C = {}^{130qtr}\!/_{10L} = 13\,{}^{qtr}\!/_{L}$	$w_C^{(i)} = 12\,{}^{qtr}\!/_{L}$ $w_C^{(ii)} = 10\,{}^{qtr}\!/_{L}$	$\omega_C^{(i)} = \frac{\left(12\,{}^{qtr}\!/_{L}\right)}{\left(13\,{}^{qtr}\!/_{L}\right)} = .923$ $\omega_C^{(ii)} = \frac{\left(10\,{}^{qtr}\!/_{L}\right)}{\left(13\,{}^{qtr}\!/_{L}\right)} = .769$	$W_C^{(i)} = \left(w_C^{(i)}\right)L = 120\ qtr$ $W_C^{(ii)} = \left(w_C^{(ii)}\right)L = 100\ qtr$	$\left(\frac{1}{\omega}\right)_C^{(i)} = 1.08$ $\left(\frac{1}{\omega}\right)_C^{(ii)} = 1.3$
D	$L_D^{(i)} = 10\,L$ $L_D^{(ii)} = 10\,L$	$\left(Y_{np}\right)_A = 120\ qtr$	$y_D = {}^{120qtr}\!/_{10L} = 12\,{}^{qtr}\!/_{L}$	$w_D^{(i)} = 11\,{}^{qtr}\!/_{L}$ $w_D^{(ii)} = 10\,{}^{qtr}\!/_{L}$	$\omega_D^{(i)} = \frac{\left(11\,{}^{qtr}\!/_{L}\right)}{\left(12\,{}^{qtr}\!/_{L}\right)} = .917$ $\omega_D^{(ii)} = \frac{\left(10\,{}^{qtr}\!/_{L}\right)}{\left(12\,{}^{qtr}\!/_{L}\right)} = .834$	$W_D^{(i)} = \left(w_D^{(i)}\right)L = 110\ qtr$ $W_D^{(ii)} = \left(w_D^{(ii)}\right)L = 100\ qtr$	$\left(\frac{1}{\omega}\right)_D^{(i)} = 1.09$ $\left(\frac{1}{\omega}\right)_D^{(ii)} = 1.2$
E	$L_E^{(i)} = 10\,L$ $L_E^{(ii)} = 10\,L$	$\left(Y_{np}\right)_A = 110\ qtr$	$y_E = {}^{110qtr}\!/_{10L} = 11\,{}^{qtr}\!/_{L}$	$w_E^{(i)} = 10\,{}^{qtr}\!/_{L}$ $w_E^{(ii)} = 9\,{}^{qtr}\!/_{L}$	$\omega_E^{(i)} = \frac{\left(10\,{}^{qtr}\!/_{L}\right)}{\left(11\,{}^{qtr}\!/_{L}\right)} = .909$ $\omega_E^{(ii)} = \frac{\left(9\,{}^{qtr}\!/_{L}\right)}{\left(11\,{}^{qtr}\!/_{L}\right)} = .818$	$W_E^{(i)} = \left(w_E^{(i)}\right)L = 100\ qtr$ $W_E^{(ii)} = \left(w_E^{(ii)}\right)L = 90\ qtr$	$\left(\frac{1}{\omega}\right)_E^{(i)} = 1.1$ $\left(\frac{1}{\omega}\right)_E^{(ii)} = 1.22$
F	$L_F^{(i)} = 10\,L$ $L_F^{(ii)} = 10\,L$	$\left(Y_{np}\right)_A = 100\ qtr$	$y_F = {}^{100qtr}\!/_{10L} = 10\,{}^{qtr}\!/_{L}$	$w_F^{(i)} = 9\,{}^{qtr}\!/_{L}$ $w_F^{(ii)} = 8\,{}^{qtr}\!/_{L}$	$\omega_F^{(i)} = \frac{\left(9\,{}^{qtr}\!/_{L}\right)}{\left(10\,{}^{qtr}\!/_{L}\right)} = .9$ $\omega_F^{(ii)} = \frac{\left(8\,{}^{qtr}\!/_{L}\right)}{\left(10\,{}^{qtr}\!/_{L}\right)} = .8$	$W_F^{(i)} = \left(w_F^{(i)}\right)L = 90\ qtr$ $W_F^{(ii)} = \left(w_F^{(ii)}\right)L = 80\ qtr$	$\left(\frac{1}{\omega}\right)_F^{(i)} = 1.11$ $\left(\frac{1}{\omega}\right)_F^{(ii)} = 1.25$
G	$L_G^{(i)} = 10\,L$	$\left(Y_{np}\right)_A = 90\ qtr$	$y_G = {}^{90qtr}\!/_{10L} = 9\,{}^{qtr}\!/_{L}$	$w_G^{(i)} = 8\,{}^{qtr}\!/_{L}$	$\omega_G^{(i)} = \frac{\left(8\,{}^{qtr}\!/_{L}\right)}{\left(9\,{}^{qtr}\!/_{L}\right)} = .89$	$W_G^{(i)} = \left(w_G^{(i)}\right)L = 80\ qtr$	$\left(\frac{1}{\omega}\right)_G^{(i)} = 1.125$

(A) Land Type	**(IX : 4)** "Rate of profits under the foregoing Circumstances" (here = rate of exploitation) $\left[r=\frac{(1-\omega)}{\omega}=\%\right]$	**(X : 5)** "Quantity of labour required to produce the wages of ten men" = number of labourers necessary to produce wage bill (paid labour-time) $\left[L_w=(\omega)L\right]$	**(XI : 6)** "Quantity of profits on the advanced of labour" = number of labourers necessary to produce profit mass (unpaid labour-time) $\left[L_r=(1-\omega)L\right]$	**(XII : 9)** "Value of Product of the Labour of Ten Men under the Circumstances Supposed" = Labour commanded by wage revenue $\left[LC_w=\left(\frac{y}{w}\right)L=\left(\frac{1}{\omega}\right)L\right]$	**(XIII)** Labour commanded by profit revenue $\left[LC_r=\left(\frac{y}{w}\right)\left(\frac{1}{r}\right)L=\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)L\right]$
A	$r_A^{(i)}=\left[\frac{(1-\omega)}{\omega}\right]_A^{(i)}=.25$ $r_A^{(ii)}=\left[\frac{(1-\omega)}{\omega}\right]_A^{(ii)}=.154$ $r_A^{(iii)}=\left[\frac{(1-\omega)}{\omega}\right]_A^{(iii)}=.50$	$(L_w)_A^{(i)}=\left(\omega_A^{(i)}\right)L\ =\ 8\,L$ $(L_w)_A^{(ii)}=\left(\omega_A^{(ii)}\right)L\ =\ 8.67\,L$ $(L_w)_A^{(iii)}=\left(\omega_A^{(iii)}\right)L\ =\ 6.67\,L$	$(L_r)_A^{(i)}=\left(1-\omega_A^{(i)}\right)L\ =\ 2\,L$ $(L_r)_A^{(ii)}=\left(1-\omega_A^{(ii)}\right)L\ =\ 1.34\,L$ $(L_r)_A^{(iii)}=\left(1-\omega_A^{(iii)}\right)L\ =\ 3.34\,L$	$(LC_w)_A^{(i)}=\left(\frac{1}{\omega}\right)_A^{(i)}L\ =\ 12.5\,L$ $(LC_w)_A^{(i)}=\left(\frac{1}{\omega}\right)_A^{(ii)}L\ =\ 11.53\,L$ $(LC_w)_A^{(i)}=\left(\frac{1}{\omega}\right)_A^{(iii)}L\ =\ 15\,L$	$(LC_r)_A^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_A^{(i)}L\ =\ 50\,L$ $(LC_r)_A^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_A^{(ii)}L\ =\ 75\,L$ $(LC_r)_A^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_A^{(iii)}L\ =\ 30\,L$
B	$r_B^{(i)}=\left[\frac{(1-\omega)}{\omega}\right]_B^{(i)}=.167$ $r_B^{(ii)}=\left[\frac{(1-\omega)}{\omega}\right]_B^{(ii)}=.273$	$(L_w)_B^{(i)}=\left(\omega_B^{(i)}\right)L\ =\ 8.57\,L$ $(L_w)_B^{(ii)}=\left(\omega_B^{(ii)}\right)L\ =\ 7.86\,L$	$(L_r)_B^{(i)}=\left(1-\omega_B^{(i)}\right)L\ =\ 1.43\,L$ $(L_r)_B^{(ii)}=\left(1-\omega_B^{(ii)}\right)L\ =\ 2.14\,L$	$(LC_w)_B^{(i)}=\left(\frac{1}{\omega}\right)_B^{(i)}L\ =\ 11.67\,L$ $(LC_w)_B^{(i)}=\left(\frac{1}{\omega}\right)_B^{(ii)}L\ =\ 12.7\,L$	$(LC_w)_B^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_B^{(i)}L\ =\ 70\,L$ $(LC_w)_B^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_B^{(ii)}L\ =\ 46.67\,L$
C	$\omega_C^{(i)}=\frac{\left(12\,{}^{qtr}/_{L}\right)}{\left(13\,{}^{qtr}/_{L}\right)}=.923$ $\omega_C^{(ii)}=\frac{\left(10\,{}^{qtr}/_{L}\right)}{\left(13\,{}^{qtr}/_{L}\right)}=.769$	$(L_w)_C^{(i)}=\left(\omega_C^{(i)}\right)L\ =\ 9.23\,L$ $(L_w)_C^{(ii)}=\left(\omega_C^{(ii)}\right)L\ =\ 7.79\,L$	$(L_r)_C^{(i)}=\left(1-\omega_C^{(i)}\right)L\ =\ .77\,L$ $(L_r)_C^{(ii)}=\left(1-\omega_C^{(ii)}\right)L\ =\ 2.3\,L$	$(LC_w)_C^{(i)}=\left(\frac{1}{\omega}\right)_C^{(i)}L\ =\ 10.8\,L$ $(LC_w)_C^{(i)}=\left(\frac{1}{\omega}\right)_C^{(ii)}L\ =\ 13\,L$	$(LC_w)_C^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_C^{(i)}L\ =\ 130\,L$ $(LC_w)_C^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_C^{(ii)}L\ =\ 43.33\,L$
D	$\omega_D^{(i)}=\frac{\left(11\,{}^{qtr}/_{L}\right)}{\left(12\,{}^{qtr}/_{L}\right)}=.917$ $\omega_D^{(ii)}=\frac{\left(10\,{}^{qtr}/_{L}\right)}{\left(12\,{}^{qtr}/_{L}\right)}=.834$	$(L_w)_D^{(i)}=\left(\omega_D^{(i)}\right)L\ =\ 9.17\,L$ $(L_w)_D^{(ii)}=\left(\omega_D^{(ii)}\right)L\ =\ 8.34\,L$	$(L_r)_D^{(i)}=\left(1-\omega_D^{(i)}\right)L\ =\ .83\,L$ $(L_r)_D^{(ii)}=\left(1-\omega_D^{(ii)}\right)L\ =\ 1.66\,L$	$(LC_w)_D^{(i)}=\left(\frac{1}{\omega}\right)_D^{(i)}L\ =\ 10.9\,L$ $(LC_w)_D^{(i)}=\left(\frac{1}{\omega}\right)_D^{(ii)}L\ =\ 12\,L$	$(LC_w)_D^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_D^{(i)}L\ =\ 120\,L$ $(LC_w)_D^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_D^{(ii)}L\ =\ 60\,L$
E	$\omega_E^{(i)}=\frac{\left(10\,{}^{qtr}/_{L}\right)}{\left(11\,{}^{qtr}/_{L}\right)}=.909$ $\omega_E^{(ii)}=\frac{\left(9\,{}^{qtr}/_{L}\right)}{\left(11\,{}^{qtr}/_{L}\right)}=.818$	$(L_w)_E^{(i)}=\left(\omega_E^{(i)}\right)L\ =\ 9.09\,L$ $(L_w)_E^{(ii)}=\left(\omega_E^{(ii)}\right)L\ =\ 8.18\,L$	$(L_r)_E^{(i)}=\left(1-\omega_E^{(i)}\right)L\ =\ .91\,L$ $(L_r)_E^{(ii)}=\left(1-\omega_E^{(ii)}\right)L\ =\ 1.82\,L$	$(LC_w)_E^{(i)}=\left(\frac{1}{\omega}\right)_E^{(i)}L\ =\ 11\,L$ $(LC_w)_E^{(i)}=\left(\frac{1}{\omega}\right)_E^{(ii)}L\ =\ 12.2\,L$	$(LC_w)_E^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_E^{(i)}L\ =\ 110\,L$ $(LC_w)_E^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_E^{(ii)}L\ =\ 55\,L$
F	$\omega_F^{(i)}=\frac{\left(9\,{}^{qtr}/_{L}\right)}{\left(10\,{}^{qtr}/_{L}\right)}=.9$ $\omega_F^{(ii)}=\frac{\left(8\,{}^{qtr}/_{L}\right)}{\left(10\,{}^{qtr}/_{L}\right)}=.8$	$(L_w)_F^{(i)}=\left(\omega_F^{(i)}\right)L\ =\ 9\,L$ $(L_w)_F^{(ii)}=\left(\omega_F^{(ii)}\right)L\ =\ 8\,L$	$(L_r)_F^{(i)}=\left(1-\omega_F^{(i)}\right)L\ =\ 1\,L$ $(L_r)_F^{(ii)}=\left(1-\omega_F^{(ii)}\right)L\ =\ 2\,L$	$(LC_w)_F^{(i)}=\left(\frac{1}{\omega}\right)_F^{(i)}L\ =\ 11.11\,L$ $(LC_w)_F^{(i)}=\left(\frac{1}{\omega}\right)_F^{(ii)}L\ =\ 12.5\,L$	$(LC_w)_F^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_F^{(i)}L\ =\ 100\,L$ $(LC_w)_F^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_F^{(ii)}L\ =\ 50\,L$
G	$\omega_G^{(i)}=\frac{\left(8\,{}^{qtr}/_{L}\right)}{\left(9\,{}^{qtr}/_{L}\right)}=.89$	$(L_w)_G^{(i)}=\left(\omega_G^{(i)}\right)L\ =\ 8.9\,L$	$(L_r)_G^{(i)}=\left(1-\omega_G^{(i)}\right)L\ =\ 1.1\,L$	$(LC_w)_G^{(i)}=\left(\frac{1}{\omega}\right)_G^{(i)}L\ =\ 11.25\,L$	$(LC_w)_G^{(i)}=\left[\left(\frac{1}{\omega}\right)\left(\frac{1}{r}\right)\right]_G^{(i)}L\ =\ 90\,L$

$L_w+L_r=10L$
(invariable labour)

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The author would like to thank Professor Garegnani for permission to quote from the Sraffa Papers.

[2] Bonar laments the fact that Malthus has been too often reviled but too little read and understood:

> "The world has no time to consult authorities, though it likes them to be within reach of consultation. When an author becomes an authority, he too often ceases to be read, and his doctrines, like the current coin, are worn by use till they lose the clear image and subscription of the user. In this way an author's name may come to suggest, not his own book, but the current version of his doctrines" (Bonar, 1924, p. 2).

This unfortunate tendency is especially strong in the discipline of economics; witness the above reference to "Sraffian" economics versus that of Sraffa. Schumpeter (1954) makes a similar lament with regard to Marshall and increasing returns:

> "Modern factual investigators who keep on discovering the existence and importance of the intervals of falling average and marginal cost in the cost curves of individual firms…and believe these findings shake the foundations of 'neo-classic' cost analysis, are really rediscovering Marshall: a striking illustration of the fact that the majority of economists *do not read*" (Schumpeter 1954, pg. 1045, note 48; emphasis in text).

[3] Shortly after Malthus's death in 1834 came Bishop Otter's (1836) glowing Memoir that accompanies the posthumous second edition of Malthus's *Principles of Political Economy,* followed closely by William Empson's (1837) favorable review of that work in the *Edinburgh Review* ; so disgusted with laudatory tone of the latter, John Ramsey McCulloch would soon after its publication never again take pen-to-paper for this Scottish Whig *Review*. Frank Whitson Fetter, son of Frank Albert Fetter, writes in 1953 that:

> "McCulloch's last contribution was in July, 1837. We have no direct evidence as to why he then terminated his long connection with the *Edinburgh Review*…However, in view of the earlier refusal of the *Edinburgh* to let McCulloch review Malthus's first edition, it is possible that so favorable a review of Malthus by Empson may have led to McCulloch's retirement as a reviewer" (Fetter, 1953, pg. 240).

Fetter's well-written and well-researched essay on the *Edinburgh* is the first in a quartet of such articles that attributes authorship (as the then-custom was anonymity) and develops the intellectual history of the economic articles in the four main British reviews of the early 19th century. Founded in 1802, the Whiggish *Edinburgh Review* is discussed in Fetter (1953); the *Quarterly Review*, a Tory rival founded in 1809 in reaction to the *Edinburgh* , is discussed in Fetter (1958a,b); in Fetter (1960) the Tory yet Scottish-Patriotic *Blackwood's Edinburgh Magazine*, founded in 1817, is discussed; and finally Fetter (1962) completes the quartet with an analysis of the Whiggish *Westminster Review* that began publication in 1824. Other notable biographies of Malthus include the above-referenced Bonar (1924), Keynes (1933 [1972]), Semmel (1963), and in more recent times James (1979) and Pullen's various contributions (1978, 1981, 1987a, 1987 b, 1987c); see especially the Introduction to the Variorum edition of Malthus's *Principles of Political Economy* (Pullen 1989; see note 12 below).

[4] In *Essays in Biography*, Keynes writes of that:

> "Ricardo, in the course of simplifying the many successive stages of his highly abstract argument, departed, necessarily and more than he himself was aware, away from the actual facts; whereas Malthus, by taking up the tale much nearer to its conclusion, had a firmer hold on what may be expected in the real world" (Keynes, 1933 [1972], p. 88)

And in the 1935 Commemoration of the centenary of Malthus's death held in the Hall of King's College, Cambridge, Keynes concludes with his "claim for Malthus a profound economic intuition and an unusual combination of keeping an open mind to the shifting picture of experience and constantly applying to its interpretation the principles of formal thought" (Keynes, 1935 [1972], p. 108). That Malthus embodied a more "empirical" approach to economic questions, as opposed to Ricardo's more abstract "philosophical" approach, has long been recognized. This observation begins first and foremost with Empson's review article:

> "Mr. Ricardo and Mr. Malthus had both minds eminently philosophical. But they differed chiefly in this, that, in looking at their subjects philosophically, the man of business [e.g. Ricardo] delighted to dwell among and follow out abstract principles, while the professor [e.g. Malthus] was constantly enquiring after practical results" (Empson, 1837 [1963], p. 240)

Empson then reminds readers that the subtitle of Malthus's *Principles of Political Economy* was "*considered with a view to their practical application*" (Id). And A. C. Whitaker, a student of J.B. Clark under whose direction wrote his Columbia Ph.D. thesis, explicitly takes this view in the published version of that work entitled *History and Criticism of the Labour Theory of*

Value (1904), crediting in the Preface Friedrich von Weiser's *Natural Value* for the "philosophical" vs. the "empirical" methodological distinction, although von Weiser did not explicitly apply it to the labour theory, which Whitaker does by association of Ricardo's philosophical "labour embodied" theory against Malthus's empirical "labour commanded" one. Malthus however does not eschew the "philosophical" approach wholesale; rather he approaches the relationship between that and the "empirical" as one of a necessary unity, what he terms in the Introduction to his *Principles* as a "just mean" between the two:

> "Aware, however, of my liability to [the] error [of drawing inferences from actual experiences, not warranted by general principles] on the one side, and to the error of not referring to sufficient experience on the other, my aim will be to pursue, as far as I am able, a just mean between the two extremes, and to approach, as near as I can, to the great object of my research – the truth" (Malthus 1820, pg. 22 in Pullen 1989, Volume I; in 1836 second edition p. 17; see Pullen 1989 Volume II pg. 14 for terminological changes in the two editions).

Keynes's biography contrasts "Ricardo…the abstract and *a priori* theorist" against "Malthus the inductive and intuitive investigator" (Keynes, 1933 [1972], p. 95). De Marchi and Sturges (1973) present four hitherto unpublished letters by Malthus to the staunch methodological inductivist William Whewell to explore the possibility that Whewell and fellow "inductivist critic" Reverend Richard Jones (who succeeded Malthus as Professor of Political Economy at East India College, Haileybury), approached Malthus "early in 1831 to test whether [he] might be willing to declare himself for the inductivist cause" as regards the critique they wanted to advance over the deductive methodology of Ricardo and the Ricardian school of Political Economy. They note that "if Whewell and Jones did hope to find an ally in Malthus, his reaction to their overtures must have been a disappointment" since Malthus "took his stand on the sensible middle ground that [*a priori*] definitions (as starting hypotheses) and empirical investigation 'act and react upon each other'" (de Marchi and Sturgis, 1973, p. 384).

[5] For example, we read in the *Grundrisse* that "Malthus on value…at least tried to proceed scientifically" (Marx 1973, p. 326) and that the Parson "senses the contradictions" in Ricardo's theory of value, "but falls flat on his face when he himself tries to develop them" (Marx 1973, p. 353). In *Theories of Surplus Value*, Marx notes that "the real contribution made by Malthus…is that he places the main emphasis on the *unequal* exchange between capital and wage labour, whereas Ricardo does not actually explain how the law of value (according to the labour-time embodied in the commodities) gives rise to the unequal exchange between capital and living labour" (Marx 1971, p. 14). Finally in *Capital* Volume I Marx observes that, "[i]n [his *Principles of Political Economy*] Malthus finally discovers, with the help of Sismondi,

the beautiful trinity of capitalist production: over-production, over-population, and over consumption. Three very delicate monsters indeed!" (Marx 1976, p. 787, note 15).

[6] See Naldi (2003, Section 3, pp. 550-554) for a discussion of two distinct conceptions of labour commanded. Scepanti and Zamagni (2005, pp. 68-72 and pp. 95-100) operationalize the labour commanded concept within the context of development of the Smith-Ricardo-Malthus analytical schema.

[7] In the first edition of Malthus's *Principles* he adopted as the measure of value the "mean between corn and labour", a position that he subsequently abandoned in favor of invariable labour alone. This he makes clear in *Measure of Value*:

> "In my last work [e.g. the first edition of his *Principles*], I thought that a mean between corn and labour might be a better measure of value than labour alone; but I am now convinced that I was wrong, and that labour alone is the true measure" (Malthus 1823 p. 24, in Porta 1992, p. 23).

On 25 May, 1823, in a discussion on Malthus's 1823 book, Trower in a letter to Ricardo writes that:

> "I am glad to see [Malthus] has abandoned his mean measure of *Corn and Labour*. No doubt he is a very candid man, and has truth alone for his object" (*Works IX*, p. 293; emphasis in original).

[8] See especially Bonar (1924), Book II, Chapter II, appropriately entitled "The Working Man", which begins precisely with the aforementioned change in Malthus's measure:

> "As the Rent and Corn pamphlets deal chiefly with Mother Earth, the tract on the Measure of Value deals chiefly with Father Work. The search for a common measure of value is not, to Malthus, a purely academic problem. He considers such a measure desirable because in any inquiry into the wealth of nations it is important to distinguish between he rise of one commodity and the fall of another" (Bonar 1924, p. 254).

[9] In Carter (2010) we develop the command notion in terms of the factors of production, capital and labour, within the context of the development of surplus and deficit industries that appear in Sraffa (1960). What we show is that as the wage share falls from unity to zero, labour commands less value and its productivity and capital commands more value than its cost. And this it is argued underscores the kernel of analysis that Sraffa was developing in his construct of surplus and deficit industries.

[10] Duncan Foley (1989, pg. 875) takes a similar approach to Robinson:

> "It is particularly striking to me that Robinson maintained her opposition to the labour theory of value so tenaciously, because she put forward very clearly what I regard as the key to the problem when she argued that the rate of surplus value, the ratio of profit income to wage income, plays the key role in determining the

potential accumulation in a capitalist economy. This is precisely the conclusion I arrived at in trying to reconstruct Marx's development of the labour theory of value by studying the conditions for the conservation of surplus value between labour value and money price accounting schemes."

[11] In *Definitions in Political Economy* (1827) Malthus makes the startling claim that his measure and that of Ricardo begin to *coincide*:

> "In the *Measure of Value Stated and Illustrated*, I considered the value of commodities as…determined by the natural and necessary conditions of supply…stated to be the accumulated and immediate labour worked up in commodities with the ordinary profits upon the whole advances for the time that they were advanced…it was certainly a very remarkable fact, that when Mr. Ricardo chose the labour worked up in commodities 'as, under many circumstances, an invariable standard,' and rejected the labour which they would ordinarily purchase…he should not have seen that the labour which a commodity will ordinarily command, *necessarily involves his own proposition*, with that addition to it which can alone make it correct; and that it us precisely because the labour which a commodity will ordinarily command measures the labour worked up in it with the addition of prodits" (Malthus, 1827, pp. 213-214; emphasis added).

Keynes writes that later in his years Malthus remarked of Ricardo:

> "I never loved anybody out of my own family so much. Our interchange of opinions was so unreserved, and the object after which we were both enquiring was so entirely the truth, and nothing else, that I cannot but think we sooner or later must have agreed" (Keynes, 1933 [1972], p. 94; also quoted in Empson, 1837 [1963], p. 261).

[12] Porta (1992, p. *x*) notes that "curiously...these Notes were not included in the Sraffa edition [of the *Works*], where they are described as 'the only considerable item' left behind". He further notes that they only receive mention by Sraffa deep in the volume on Biographical Miscellany (*Works X*): "They are, in fact, only described in vol. X, app. B, containing the final survey of Ricardo's manuscripts, which appeared four years later, in 1955" (Id., note 6).

[13] The *Measure of Value* began as an effort for Malthus to revise his *Principles of Political Economy* published in 1820 (see *Works II*, p. *xii*; Pullen 1989, pp. *xlii-xliv*). The second edition was not published until after Malthus's death, and significant changes occur in the two editions, not all of which can be unequivocally be distinguished between Malthus's intended revisions as that of the anonymous editor, who Pullen (1978, 1989 pp. *lxi-lxiv*) convincingly conjectures to be

John Cazenove. The variorum edition of Malthus's *Principles* edited by Pullen (1989) meticulously documents the changes in the two editions. In his well-written introduction to that work, Pullen (1989) notes that

> "The *Measure of Value* could be described as the least successful of Malthus's publications. It was an attempt to prove that the quantity of labour commanded in exchange is the only measure of the 'natural value' of commodities, and that the value of labour is invariable. If he had been content to argue that the quantity of labour commanded is a useful and approximate measure of value, few economists would have disagreed, even if index numbers measuring general purchasing power had been in existence. But his view that labour commanded is the sole measure and an invariable measure of value found little support amongst his contemporaries, who saw his argument as unconvincing or tautologous" (pg. *xliv*).

Pullen then cites Samuel Bailey's (1825) critique, as well as de Quincey's (1824) critique. On Malthus's 1825 lecture, published in 1829, Pullen remarks that:

> "It was essentially a repetition of the arguments in the *Measure of Value*, 1823, and was equally unconvincing" (pg. *li*).

For his part, Marx thought little on the merits of Malthus's illustration in the *Measure of Value*, calling it

> "a real example of feeble-minded thought, which winds its way in a causistical and self-stupefying manner through its own inner confusion, and whose difficult, clumsy style leaves the unprejudiced and incompetent reader with the impression that the difficulty of making sense out of the confusion does not lie in the contradiction between confusion and clarity, but in a lack of understanding on the part of the reader" (Marx, *TSV*, Part III, pg. 25)

[14] This appears on page 34 of the Porta (1992) reproduction of Malthus's text and Ricardo's notes.

[15] The Roman numeral represents the column number in our extensions and the arithmetic number the column number in Malthus (see Appendix B).

[16] This formal model is also discussed in the groundbreaking works of Costabile (1983) and Tosato (1985), both of which deserve further treatment which is unfortunately beyond the scope of the present work.

[17] Marx is very clear about the idea that capital by its nature is nothing but the command of unpaid labour. This we find in *Capital* Volume I in the very last sentence of Chapter 18 Different Formulae for the Rate of Surplus Value, which completes Part Five on the Production upon which we embark onto Part Six on Wages:

"Capital, therefore, is not only the command over labour, as Adam Smith thought. All surplus-value, whatever particular form (profit, interest, or rent) it may subsequently crystallize into, is in its substance the materialization of unpaid labour-time. The secret of the self-valorization of capital resolves itself into the fact that it has at its disposal a definite quantity of the unpaid labour of other people" (Marx, 1976, p. 672).

[18] This represents a replica of the original table found in Malthus [1823, p. 38], here taken from Porta (1992, p. 34). Note that the italicized definitions and equations are our own from the extensions in the first and second rows. The only column we do not replicate in our extensions is Malthus's Column 8, which is essentially a quantity index wherein the fundamental conditions of labour commanded are normalized such that each capital on the various land-types produces 100 quarters of corn.

[19] Malthus does not identity these 14 distinct cases in this manner, and although the reproduction of the Table in de Quincey's (1824) Sixth Dialogue does label the distinct case from alpha to xi, he (De Quincey) does not order each case according fertility in the same manner as in the present table.

[20] Different capitals are denoted by the respective superscripts on the wage; i.e. $w_A^{(i)}$ denotes the first capital employed on land of fertility A, $w_A^{(ii)}$ the second capital employed on land of fertility A, etc.