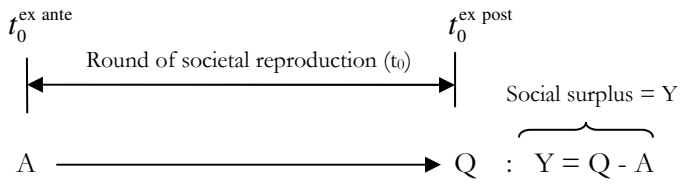
A simple model of the surplus approach to value, distribution, and growth[1]

The Social Surplus Defined

The "social surplus" is defined generally as a quantum of (new) wealth, value, and/or product over and above that necessary for a social system to reproduce itself. It can be conceived as the residual after the societal necessaries have been properly accounted for and deducted out of total end-of-period gross output. Letting A[2] stand for these societal necessaries at the beginning of the round of social reproduction (t_0) and Q stand for total gross output emergent at the end of this round, the surplus (Y) can be seen as the difference between Q and A. This simplistic interpretation is depicted in Figure 1:

Figure 1: Societal reproduction and the social surplus as a residual



The surplus component has been alternatively termed the *net product*, *value added*, *gross profits*, and *shares of remunerated national income*. It has been defined in different contexts as both *excluding* as well as *including* wages. It has been conceived as the fund from which distributed revenues partake, and is also the source of capital accumulation and growth in subsequent rounds (t_{+1} ; t_{+2}, …) of social reproduction.

The existence of surplus so defined is the hallmark of modern capitalist socio-economic systems; the net productivity associated with this mode of production is enormous. This is not to suggest that in

earlier historical epochs a social surplus did not exist. From the moment when human civilization emerged out of the hunting and gathering stage the social surplus was the source of net wealth, it was what enabled the construction of Babylonian hanging gardens and Egyptian pyramids, etc. However what is indisputable is that the growth in net wealth – viz. the *surplus* - became exponential in capitalistically-oriented systems of production and distribution, dwarfing the surpluses engendered in any previous socio-economic system. Economic theory aims, in part, to explain the phenomenal ability of capitalism to generate and accumulate large social surpluses.

We speak of two broad approaches to the social surplus in capitalistic and market oriented systems, each couched within one of two respective theoretical paradigms of value and distribution (see Garegnani 1984). The more recent or "modern" of the two is the marginalist or neoclassical approach to value and distribution. This approach remains the dominant one in economic theory. Net productivity here is recognized, however it is *not* referred to as a "surplus"; "surplus" in this approach is conceived *not* in terms of net wealth creation, but rather in terms of "extra" satisfaction gained, seen in the concepts of consumer surplus and producer surplus – i.e. quanta of "satisfaction" that accrue to the respective individual "demanders" and "suppliers". The opposing forces of demand and supply especially of the "factors of production" seen to "naturally" (i.e. "imperfections" cast aside) come into equilibrium such that remuneration of each factor corresponds to its (marginal) productivity. Increases in social net wealth are due to the (marginal) productivity of each factor, especially that of capital (human, physical, or entrepreneurial). Wages accrue to labor and profits accrue to the owners of the various forms of capital. Euler's theorem ensures that, under perfect competition, each factor is paid according to its contribution to net-wealth creation. This theory conceives the market the societal expression of human nature, and as a fundamentally harmonious mechanism.

On the other hand, the surplus approach to value and distribution represents the older tradition in political economy the origins of which we can trace to the circular flow of the Francois Quesnay's famous *Tableau Economique*. Despite the feudal residue of its treatment of agriculture as the sole "productive" sector – i.e. as the only source of the surplus product – on a fundamental plane the *Tableau* contains insights that remain relevant. Modern developments to the surplus approach begin with the publication of Piero Sraffa's (1960) thin but quite terse and somewhat cryptic book *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory*. Eschewing completely the marginalist approach to value and distribution (one neoclassical reviewer laments that there is no literature in the book cited after 1914)[3], Sraffa lays the foundations for what Ronald Meek has coined the "rehabilitation" the Classical approach to value and distribution.[4] It was Sraffa's student Pierangelo Garegnani who most precisely articulated certain characteristic features of the surplus approach, what he terms the "core" in the surplus theories. For Garegnani and subsequent theorists in this tradition, the "surplus" by definition consists of the non-wage portion of the share of national income:

> Besides the necessary replacement of the means of production [it] included the subsistence of the agricultural labourers…[t]he fact that the subsistence of workers was considered necessary for reproduction established a direct link between [the] analysis of social reproduction and that of the division of the product among the classes society is divided…[i]n principle [the] way of determining the non-wage incomes is simple. Three sets of circumstanced are assumed to be known prior to this determination…(i) the real wage…; (ii) the social product: that is the output of the commodities produced in the year; (iii) the technical conditions of the production" (Garegnani 1987, pg. 560).

The question of whether or not wages should be included in the surplus is a major analytical point of debate within the surplus approach to which we return below.

The Physiocratic basis of the notion of "Surplus" in the Classical economists, Marx, and Sraffa

We trace the origin of the classical surplus concept to the Physiocrats and the *Tableau*. The groundbreaking character of Physiocratic thought cannot be underestimated. Writing in pre-revolutionary France, Quesnay, his close disciple Mirabeau, and other adherents to the Physiocratic school developed the first formal economic model of an integrated economic system. At the heart of the Physiocratic doctrine was the concept of the *produit net*, or *net product*. As Ronald Meek (1962 p. 19) has put it, "the Physiocrats…endeavoured to discover some key variable…causing an expansion or contraction…in the general level of economic activity. The variable which they hit upon was the capacity of agriculture to yield a 'net product', i.e. a disposable surplus over necessary cost". The Physiocrats expressed the social surplus within the "feudal shell" of agricultural production. Only the agricultural sector was deemed "productive" in the sense that it alone was thought to yield the excess of product above cost, hence they deemed the social surplus the "gift of nature". The other two sectors in the Physiocratic framework were to be "sterile" in the sense that no new net product was created by this activity; (i) the manufacture sector was conceived as "sterile" in the sense that output was produced, the "value" of which was seen as the same magnitude as that of the inputs; and (ii) the landlords were not even "sterile" in the sense that they represent pure (unproductive) consumption without any equivalent exchange.

Moving to developments by economists on the British Isles, one of Adam Smith's many original contributions to the then-budding science of political economy was precisely the generalization of

the Physiocratic "gift" from agricultural production to manufactured production proper. Rejecting the old mercantilist notion that national wealth consisted of hoarded precious metals and the necessity of favorable terms of trade manifest from protectionist trade prescriptions, for Smith the wealth of nations was instead expressed in the net productivity of a nations' industry and workforce. We may perhaps conceive this as the "gift of net productivity" that modern surplus producing society is able to enjoy.

The classical economists proper, specifically David Ricardo and Thomas Robert Malthus, also conceived of an economic system within a surplus approach paradigm. For his part Ricardo placed great emphasis not only on net productivity, but even more importantly on the distribution of this net productivity to the three original "classes of the community". Here we find the primal role given to the distribution of the "produce of the earth" as the remunerated revenues of wages to the class of laborers, rents to the class of landlords, and profits to the class of capitalists. Malthus too worked within the surplus approach paradigm, and the fundamental differences between him and Ricardo revolved not around the efficacy of the approach *as such* but rather on certain nuances within this approach such as the correct measure of value and the possibility and implications of overproduction/underconsumption (the debate on Say's law).

The developments by Karl Marx, although strictly speaking a *critique* of then-extant (bourgeois) political economy, also remain within the confines of the surplus approach paradigm. Marx's developments are actually less clear-cut, and it is here we find a different interpretation of the notion of "surplus". Whereas with the Physiocrats, Smith, Ricardo, and Malthus, the surplus is in the main conceived as surplus *product* – i.e. a surplus quantum of output over and above the necessary conditions of social reproduction, in Marx we encounter quite explicitly the idea that this surplus product in fact is the material expression of *surplus value* explicitly conceived as *unpaid labor time*. This

is to say, for Marx the idea of a "social surplus" lay squarely in the exploitative nature of a skewed class society. Here we can begin to distinguish the Sraffian surplus from the Marxian surplus. The Sraffian surplus by definition encompasses the entire net product, both the wage portion as well as the profit portion. The Marxian surplus, by contrast, includes only the profit portion, the wage portion being relegated to the realm of necessaries. Sraffa himself struggled with this difference with Marx, as we shall see below, and was able finally to resolve this with the distinction between the subsistence vs. the surplus wage.[5]

Turning to Sraffa, the surplus approach to value and distribution in its modern version can be traced to the interpretation of the Italian Cambridge economist. When one considers the peculiar mode of exposition that Sraffa ultimately chose for his book, as expressed in the first five chapters, we find that Sraffa adopts a "physicalist" approach to the transition to capitalist commodity production. The term "physicalist" refers to the development of the model using the physical structure of inputs and outputs measured in given quantities of use-values, and the surplus-producing model is differentiated only by the quantum of "extra" output at the end of the production round.[6] Sraffa first posits an "ideal type" system of simple commodity production found in Chapter I's production for subsistence model where no surplus product emerges. This represents the physicalist analogue of Adam Smith's "early and rude state"[7] such that the labor requirements are not made explicit and instead are regulated as analogous to fodder for animals and hence included within the means of production requirements.[8] Sraffa gives no mention is given to the implicit unitary wage share of this system; we are only told that this "simple society…produces just enough to maintain itself", and to illustrate Sraffa gives us a simple numeric example that shows the summed economy-wide inputs "necessary" for production are exactly equal to the quantity of resulting outputs. The model for capitalist commodity production is presented in Chapter II's production with a surplus model. The surplus production conceived here is the case where for at least one commodity more output is

produced than is "necessary" as inputs for systemic production. Here the surplus is conceived as surplus *product*, resulting in "extra" output left over after the conditions of production have been replaced. The numeric example in the two chapters highlighting this "physicalist" aspect is seen quite clearly when placed side-by-side:

Table 1: Production for subsistence and production with a surplus numeric examples in Sraffa (1960)

Production for Subsistence (Sraffa 1960, Ch. I, p. 3)	**Production with a Surplus (Sraffa 1960, Ch. II, p. 7)**
$\begin{pmatrix} 280 \text{ qr. wheat} & : 12 \text{ t. iron} & \rightarrow & 400 \text{ qr. wheat} \\ 120 \text{ qr. wheat} & : 8 \text{ t. iron} & \rightarrow & 20 \text{ t. iron} \end{pmatrix}$	$\begin{pmatrix} 280 \text{ qr. wheat} & : 12 \text{ t. iron} & \rightarrow & 575 \text{ qr. wheat} \\ 120 \text{ qr. wheat} & : 8 \text{ t. iron} & \rightarrow & 20 \text{ t. iron} \end{pmatrix}$
$\sum = (400 \text{ qr. wheat} : 20 \text{ t. iron})$	$\sum = (400 \text{ qr. wheat} : 20 \text{ t. iron})$

The summed means of production requirements for each system is shown in the penultimate row. The physical structure of production on the input-side is exactly the same, the only difference being that in surplus-producing society we now have the addition of an extra quantum of output, here to the tune of 175 quarters of wheat. Because Sraffa explicitly adopts the notion of the "subsistence wage," this "extra" quantity of wheat is available for distribution *as profit*. Here Sraffa directly identifies the surplus product with *profit*. The distribution of this surplus-qua-profit to the owners of the means of production is strictly along the capitalistic lines of a uniform rate of return on the capital advanced in each industry. It is in this way that the owners of the capital in the iron industry, although they produce no physical surplus[9], nonetheless share in the gains "produced" by the wheat industry in that the exchange values of both wheat *and* iron will include the general rate of profit (r*).

Surplus as the value-added and the choice of the wage

Both Sraffa's and Marx's conceptions of the surplus conceive of the surplus as the residual remaining once the so-called "necessaries" have been deducted out of the total gross product. But Sraffa in archival notes himself questions exactly what constitutes "necessary", such that the very definition of the "surplus" turns on this definition. As early as 1928 we find Sraffa deeply concerned with this issue. In a five page document entitled "Surplus Product", Sraffa remarks that

> The study of the 'surplus product' is the true object of economics: the great difficulty of the matter is that this object either vanishes or remains unexplained. It is a typical problem to be handled dialectically. This notion is connected with that of 'necessity', and 'necessity' has only a definite meaning from a given point of view, which must be explicitly stated and adhered to consistently. The surplus product goes to expenses which are not 'necessary' for producing a given commodity (D3/12/6/161 : 1).[10]

The crux of this matter revolves around the conceptions of the wage, since there is no ambiguity about the "necessary" character of the means of production requirements. Sraffa in somewhat of a fudge introduces two distinct notions of the wage, the "subsistence" portion plus the "surplus portion". Taken literally, this idea of the wage is expressed in the following equation:

Total wage to workers = subsistence portion of wage + surplus portion of wage

Sraffa himself treats the subsistence portion of the wage as part of the means of production, which are in a sense "augmented" by the subsistence portion of the wage. Sraffa speaks of this "double character of the wage" (Sraffa 1960 § 8: 9), but in the end refrains from employing it and instead opts to treat "the whole wage as variable" (Sraffa, 1960: 10). It is our contention that Sraffa's abstraction from the subsistence portion of the wage was done so not so much for convenience but

rather because of the analytical difficulties associated with the subsistence wage concept. Adoption of the share approach certainly is articulated in the important Majorca Draft of March 1955, which was Sraffa's first attempt to pen a working draft of the first part of his book on single product industries while holed-up in the luxurious Palma Hotel on the island of Majorca in early 1955.[11] From that document we read that:

> Hitherto we have regarded wages as being composed of the necessary subsistence for the workers and thereby being on the same footing as the fuel for the engines or fodder for the working animals. Wages however partake of a double nature and while always containing the element of subsistence they may, in certain social conditions, contain secure also a share of the surplus product. ~~It thus becomes necessary to treat wages~~ And it is indeed the stress and strain arising from this possibility that has given economic theory its shape. (Sraffa Papers, D3/12/52/6)

Notice the importance that Sraffa gives to the so-called "surplus component" of the wage – namely that it is the "stress and strain" associated with the struggle of workers to secure a portion of the surplus product as opposed to capital "that has given economic theory its shape". And it is here that we follow Sraffa and conceive of the "surplus product" as the magnitude of product and/or value over and above the necessary means of production requirements, in other words, the net product, the division of which falls into wage vs. profit revenues. This surplus represents the newly created wealth of a productive economic system, and following the Classical and Marxian tradition, the source of this newly created wealth lies solely in the productive power of living labor. In Carter (2011a) we have termed the notion that living labor is the source of newly created (net) wealth the *fundamental economic normalization*. This certainly is made quite explicit in Sraffa (1960) in terms of the setting of the total value of living labor in paragraph 10 to the value of the net product in paragraph

12, both of which are normalized as unity (see Bellofiore 2008, 2010). As early as December 1942, Sraffa remarked that "the net product as a whole is always produced by L" (D3/12/29/18). And in both the Majorca Draft and in *Production of Commodities*, Sraffa eschews the bundle "subsistence" approach in favor of the share "surplus" approach to the wage, and in doing so sets the stage for setting both the total value of living labor and the value of the net product equal to unity. This is to say, the formal model that Sraffa develops in his economic system is one where the surplus product itself is associated with the explicit inclusion of the requirements of living labor. In the Majorca Draft, immediately following the above quote regarding the "stress and strain", we read the interesting passage that:

> It therefore becomes necessary to ~~treat transfer wages from~~ treat wages, like the rate of profits, as one of the variables in the system. We shall therefore have to represent explicitly the quantity of labour employed in each industry – ~~instead of~~ in the place of the quantities of subsistence we have hitherto rolled up with the raw material…We shall designate the wage per unit of labour as w, this is ~~this will be~~ a price to be determined like all other prices ~~by the solution of equations, and which is expressed~~ in terms of the commodity which is chosen as a ~~measure~~ (standard) ~~of prices~~. The unit of labour being the annual social labour, w will at the same time represent the fraction of the national income that goes to wages (it will be the proportionate wage of Ricardo)' (D3/12/52/6-7).

The parallel passage of this first draft appears precisely in the famous paragraph 10 in *Production of Commodities*. It is important to emphasize that for Sraffa the wage here represents the surplus wage, namely the *fraction* of total national income or surplus product, the latter quantity being produced by the total annual social labor. This essentially involves the adoption of the share approach to the wage, because here the wage rate, w, can be directly assessed to the quantum of living labor, as

opposed to representing a "given" quantity of a subsistence bundle. On this latter point Sraffa continues:

> The double nature of wages, as means of production and as a share in the surplus, gives rise to the question whether the whole of wages should be included in the w; or only the 'surplus' part of them, while the subsistence part is left in its previous shape as raw material…There is much to be said for either method. The latter of separation is in many ways more reasonable, for it represents by a constant the quantity that is unchanging and by a variable the quantity that can be more or less according to circumstances. It is the view ~~of A. Smith and Ricardo~~ of the classical economists, who did not regard wages as part of the national income, but as being advanced out of capital, although they did admit that in some cases they might succeed in getting part of the surplus. It ~~would also~~ has also the advantage of including the necessary wage in the year's advances which must be multiplied by the factor $1 + r$, while leaving outside the part which must come out of the product at the end of the year" (D3/12/53/7-8).

Sraffa however decides *against* this bundle *ex ante* approach to the subsistence wage:

> Much as there is to recommend it in theory this method would give rise to some difficulty in practical application, if such had ever to be attempted; for the segregation of the commodities composing the two parts of the wages while no doubt feasible in the early stages of society, would meet in modern conditions with 'insurmountable obstacles'[12] (the attempt to overcome them by a division of the 'value' of the wage would create more trouble than it overcomes)" (D3/12/52/8);

Thus Sraffa settles on the share approach to the wage:

> The other {share} method has much less logic to recommend it; but it has the decided advantage of being in conformity with the current wage, in that *it treats the wage as a single unit*, and it includes the whole of it as part of the national income; and it is the distribution of this (rather than that of the less inclusive 'National Net Revenue' of A. Smith) which we shall have to discuss. It implies also accepting the view that wages are paid entirely out of the annual product, instead of being advanced…In what follows we shall therefore include the whole wage under w" (D3/12/52/8; emphasis added).

Notice how in these latter two passages Sraffa explicitly introduces historical context: the "early stages of society" - we conjecture that this is in reference to Adam Smith's "early and rude state", i.e. the no-surplus production model - are compatible with the wage conceived as a bundle; whereas in "modern conditions" – we conjecture the surplus production model - the wage conceived as a share becomes much more relevant. And it is very interesting to note why: because in "modern conditions" the wage is "treat[ed] as a single unit". Clearly Sraffa is referring here to the *money* wage[13], or at least to a wage that is paid to the worker as a single unit of *numeraire* per hour, which in the system of equations is multiplied only by the quantum of living labor added, *L*. Hence by conceiving the wage as a share of the annual product, Sraffa was able to avoid in general the "troubles" of the composition of the basket of worker consumption goods that arise when workers do not purchase with their wage the required quantities of such goods. The share approach to the wage also avoids the thorny issue of the organic composition of the wage-good sector.[14] The juxtaposition of the bundle vs. the share approach to the wage in the context of Sraffa was first brought to light by John Eatwell in two important articles written in the mid-1970s (Eatwell 1974, 1975), where we read that "necessary labor time could be defined in two ways: (A) as the value of 'the sum of money v expended upon labor-power', in effect as the *share* of wages in the value of the output, and (B) as 'the value of (the) means of subsistence', that is, as the value of the commodities

comprising the real wage" (Eatwell 1974, p. 299; emphasis in text, and inserted quotation from *Capital*, Volume I). Although revisiting the scope and character of this debate is beyond the scope of our present task, that Eatwell faced vehement criticism by both and Marxist and Sraffian alike is beyond dispute, and we think quite misplaced.[15]

Hence we have come into marked distinction between the development of the surplus approach to value and distribution advanced in the present essay versus that development by Professor Garegnani. For Professor Garegnani (1984, p. 311), one of the "characteristic premises" of the surplus approach is that the "the real wage and the social product are given when determining the rate of profit and relative prices". Hence the "surplus" here is exclusively that of the non-wage portion of the value-added. By contrast, the approach adopted here conceives of the "surplus" as the entire value-added, from which wages *as well as* profits are remunerated to respective owners of "factors" of production. We have attempted to show through archival evidence that our approach is completely compatible with Sraffa's own thinking, and in this capacity can in at least a small degree vindicate Eatwell's (1974, 1975) original framing of the question related to this issue.

The "Value Theory of Labour"

Under these assumptions we can now begin to formalize our model of single product industries.[16] We begin first with a system of surplus production that has abstracted from the particular distribution of the surplus-*qua*- value-added between wage vs. profit revenue. In archival notes, Sraffa refers to this as the "value theory of labour":

> 21.2.1955
>
> In the dust raised by the controversies on the Labour Theory of Value, a valuable interesting aspect has been overlooked, or what be called the Value theory of Labour…For, whatever

> disputes there may be about the determination of value by the quantity of labour, there can be no doubt (nobody doubts so far as I know) that the value of a commodity (its price for r = 0) determines (i.e. measures) the quantity of labour which directly or indirectly has entered into its production. (Sraffa Papers D3/12/44/3)

This is an economic system conceived when the rate of profit is zero and all net productis distributed to workers in the form of what Pasinetti (1977, p. 122) calls the "complete wage rate" (see also Carter 2011a). In terms of a simple two-commodity model of single product industries, Sraffa's "value theory of labour" can be expressed as the following system of equations:

$$\begin{aligned} A_{11}p_1^0 + A_{21}p_2^0 + L_1 w^0 &= Q_1 p_1^0 \\ A_{12}p_1^0 + A_{22}p_2^0 + L_2 w^0 &= Q_2 p_2^0 \end{aligned} \qquad (1)$$

The physical structure of production represents known quantities, represented by the ith commodity input requirement for the jth industry $\left(A_{ij}\right)$ as well as the jth industry's direct labor requirement $\left(L_j\right)$ necessary for the jth industry's output $\left(Q_j\right)$. This represents a system of two equations in three unknowns, the respective prices of each commodity $\left(p_1^0, p_2^0\right)$ and the "complete" wage $\left(w^0\right)$. To close this system we make the assumption of an "unassisted" commodity-money sector, which we call "gold", the physical measure of which is directly in terms of currency units (i.e. "$"):

$$\begin{aligned} L_{gold} w^0 &= Q_{gold} p_{gold}^0 \\ p_{gold}^0 &= 1 \end{aligned} \qquad (2)$$

All price variables have been subscripted with zeroes to indicate that they represent what in the Marxian literature is referred to as direct prices (see for example Shaikh 1977), i.e. prices proportional to values.

The concept of the "complete" wage figures heavily in the following analysis and thus requires some further comment. Pasinetti (1977, pg. 122) coins the particular term in the context of the introduction of a no-distribution "ideal system: of production:

> An "ideal" system of prices, as understood, for example, by the "Ricardian socialists" (who had claimed at the beginning of the 19th century that the whole net product of an economic system ought to go to the workers), might be $\boldsymbol{pA} + w\boldsymbol{a_n} = \boldsymbol{p}$.[17] This is a linear system of (n – 1) equations. It determines (n – 2) relative prices and a wage rate *which absorbs the entire net product of the system*. This [is] regarded as the "maximum" wage rate…since it corresponds to a profit rate of zero. We may call it the "ideal" wage rate here, or, from a different point of view, the "complete" wage rate" (emphasis added).

In our interpretation, the "complete" wage rate is not merely one of "complete" distribution of the net product to workers; more importantly in it represents the relation of the *net productivity of labor*. – i.e. it is a relation not only of "complete" remuneration but also net productivity (see Carter 2011a).

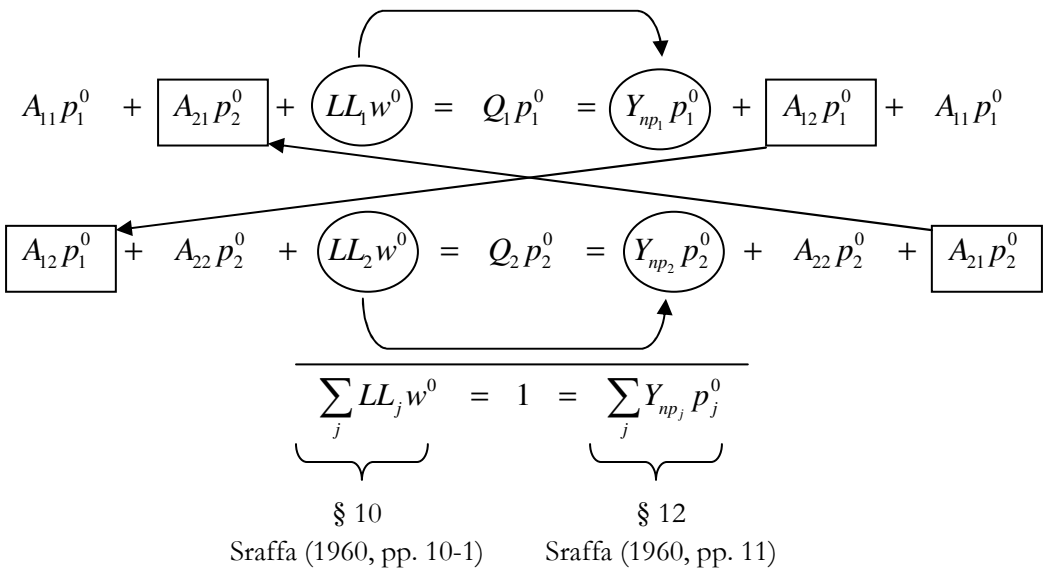
In this system, a net surplus product is assumed to be produced in the sense that the sum of the commodity input requirements for each industry are of a lesser quantity that the resulting gross output of that industry, i.e. $\sum_{i=1}^{2} A_{ij} < Q_j$. We assume that this is the case for each industry, hence both industries are fertile, but the only requirement for a surplus producing system is that at least one industry is fertile, with the other possibly being sterile.[18] Accordingly in terms of sheer physical use-values, the total gross product of each industry can be broken down into the following commodity structure:

Industry 1 = Wheat: $A_{11} + A_{12} + Y_{np_1} = Q_1$

Industry 2 = Iron : $A_{21} + A_{22} + Y_{np_1} = Q_2$

We have already seen that Sraffa with the adoption of the pure surplus wage model necessarily links the total value added by living labor to the value of the net product. Hence for a two-commodity surplus-producing system of basic goods, the commodity and industry structures are related in the following manner:

Figure 2: Integrated structure of two commodity basic system



In this figure the surplus product is represented by the total value of the net product $\left(\sum Y_{np_j} p_j^0\right)$, which in the price structure is related directly to the value-added by workers whose labor is assessed at the "complete" wage rate. At the level of the industry the value-added by workers is in general not equal to the value of the net product; this would only be the case when equal organic compositions of capital prevail throughout. However as Sraffa does in paragraphs 10 and 12 of his book, the system is so normalized such that in the aggregate these respective values do coincide.

Pasinietti's "complete" wage rate is very much like the monetary expression of labor-time (MELT) concept developed independently in the early 1980s by Gerard Dumenil (1982-83) and Duncan Foley (1982) that appears in the New Interpretation literature.[19] The major difference between Sraffa's normalization and that of the DF framework is that the latter do not explicitly set each quanta equal to unity. The MELT represents the "unit money-price" at which living labor is net-productive, and is measured in "dollars" per unit labor. As with the Dumenil-Foley interpretation, this is simply the inverse of the "value of money" concept (and vice versa) found in Marx:

$$\begin{aligned} w^0 &= \$/unit_L = \$/HR = \left(\lambda_\$^{-1}\right) \approx MELT \\ \lambda_\$ &= unit_L/\$ = HR/\$ = \left(p_{LL}^{-1}\right) \end{aligned} \tag{3}$$

The net productivity of the surplus-producing system is constituted by the system's net product, which under our assumption normalizes the value of living labor's productivity to the value of the newly created product, conceived here as the fruits of the productive laborers.

Consider now the analogue of the capital-labor ratio that Sraffa (1960) employs, which we term the labor the means of production ration (LMP). Taken right out of Sraffa (1960, Chapter III), the LMP ratio defined as the value of the living labor added divided by the value of the means of production requirements:

$$\text{Labor to Means of Production Ratio} = LMP = \frac{w^0 LL}{pA} = \frac{pY_{np}}{pA} = R = \text{maximum rate of profit} \tag{4}$$

It is also equal to the value of the net product divided by the value of the means of production, both of which come to coincide with the maximum rate of profits (R). The normalization consistent with the New Interpretation such that the value-added by living labor is set equal to the value of the net product is retained in what follows.

Functional income distribution and growth: Further development of the surplus component

Having posited the fundamental economic normalization such that the value-added by living labor is equal to the value of the net product, both of which are under our assumption equal to the surplus produced in the economic system, we now turn to the question of how this surplus is distributed as revenue and to what ends the distributed revenue is utilized. One of the most developed expositions of the relationship between functional income distribution and economic growth from the perspective of the surplus approach to value and distribution appears in Duncan Foley and Thomas R. Michl's (1999) *Growth and Distribution*, and what follows in the present essay is greatly informed by this exposition. There the accounting relationship between the surplus conceived as forms of distributed revenues (national income) is juxtaposed against the surplus conceived as forms of aggregate expenditures. This gives rise to the surplus-product accounting T-table where the value-added by productive living labor (Y) is expressed in terms of the income account of aggregate income divided into wage and profit distributive revenues and the output account of aggregate expenditures divided into consumption and investment.

Table 2: Surplus Product T-Table of Income and Output Accounts

Income Account (Distributed Aggregate Revenues)	**Output Account (Aggregate Expenditure)**
Surplus product = Y = W + Z	Surplus product = Y = C + I

Table 2 shows the two different aspects of the social surplus = value added = newly created wealth. On the income account side of the ledger, this newly created wealth is divided into wage (W) and profit (Z) revenues. The aggregate levels of wage and profit revenue can be conceived as the

respective income shares (ω = wage share and π = (1 – ω) = profit share) multiplied by the value-added by productive labor:

$$W = \omega Y$$

$$Z = \pi Y = (1 - \omega)Y$$

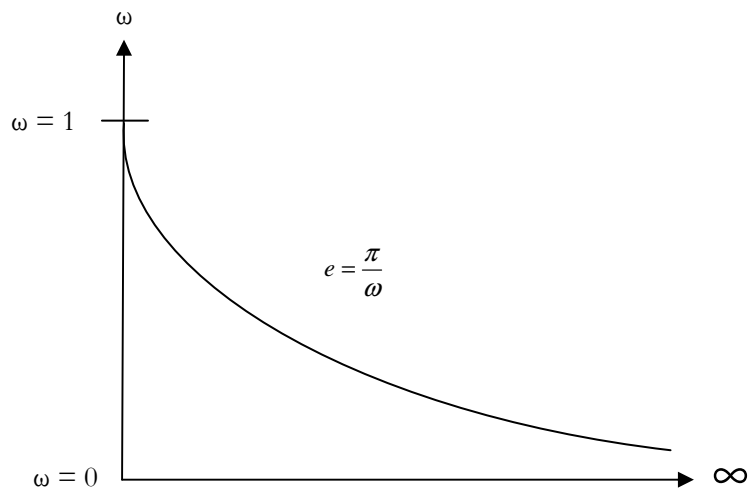
The determination of the respective income shares is - as in Sraffa (1960) - for the present purposes left open. The system can be closed on the wage-side or the profit side of the income ledger and what follows still remain equally valid. What is clear is that irrespective of the mechanisms of closure of the distribution parameter, at the level of abstraction considered here the ratio of distributive shares can be construed as an index of the Marxian rate of exploitation. The idea of the rate of exploitation as a ratio of distributive shares is the approach taken by Joan Robinson, who despite her rejection of the labor theory of value, in the Preface to the Second Edition of *An Essay on Marxian Economics* locates "the theory of exploitation" with "the theory of the distribution of the net product of industry between wages and profits" (Robinson, 1966: *vii*).

Each distributive share is in inverse relation to the other and both are upper-bounded by unity and lower-bounded by zero. In terms of the ratio of distributive shares, which we chose to call the rate of exploitation (e_j; where j = value of wage share) , the lower limit is zero and corresponds to a wage share of unity (ω = 1) and a profit share of zero (π = 0) whereas the upper limit is infinite and corresponds to a wage share of zero (ω = 0) and a profit share of unity (π = 1):

$$\begin{aligned} e_{\omega=1} &= \frac{\pi_{\omega=1}}{\omega_{\omega=1}} = \frac{0}{1} = 0 \\ e_{\omega=0} &= \frac{\pi_{\omega=0}}{\omega_{\omega=0}} = \frac{1}{0} = \infty \end{aligned} \tag{5}$$

We can depict this in terms of the following graph:

Figure 3: Rate of exploitation as the ratio of distributive income shares



When the rate of exploitation is greater than zero, the net product of industry or surplus is divided between wage and profit revenue, each of which is expressed as the respective distributive share multiplied by the value created by productive living labor. Making explicit the means of production requirements (A), the rate of profit (r) emerges defined as the ratio of profits to the value of the capital advanced. Abstracting from depreciation, the rate of profit is given by:

$$r_j = \frac{Z_j}{pA} \tag{6}$$

Given the static nature of inquiry, within each technology the rate of profit will vary with changes in the wage share (ω = j) such that when the latter is unity, the rate of profit is zero, and when the latter is zero, the rate of profit is at its maximum value R. It is important to note that the maximum rate of profit is associated with a rate of exploitation that is undefined – i.e. is infinity. Here the entire value-added is unpaid and is remunerated to the owners of capital as pure profits.

The question now turns on the relationship between the distributive revenues, wage revenue and profit revenue, and the subsequent uses of each of these revenue forms. In order to keep the analysis as simple as possible, we introduce the standard assumption in many theories of growth that workers' aggregate revenues are completely consumed; that is to say workers do not save. Hence all savings revenues subsequently mapped to investment will in this simple model come wholly out of profit revenues.

The rate of exploitation defined above, the determination of which is left open for the present purposes, allows for the division of net (surplus) productivity into the distributive revenues wages (W) and profits (Z), each of which will be equal to the respective income share multiplied by net (surplus) product (Y):

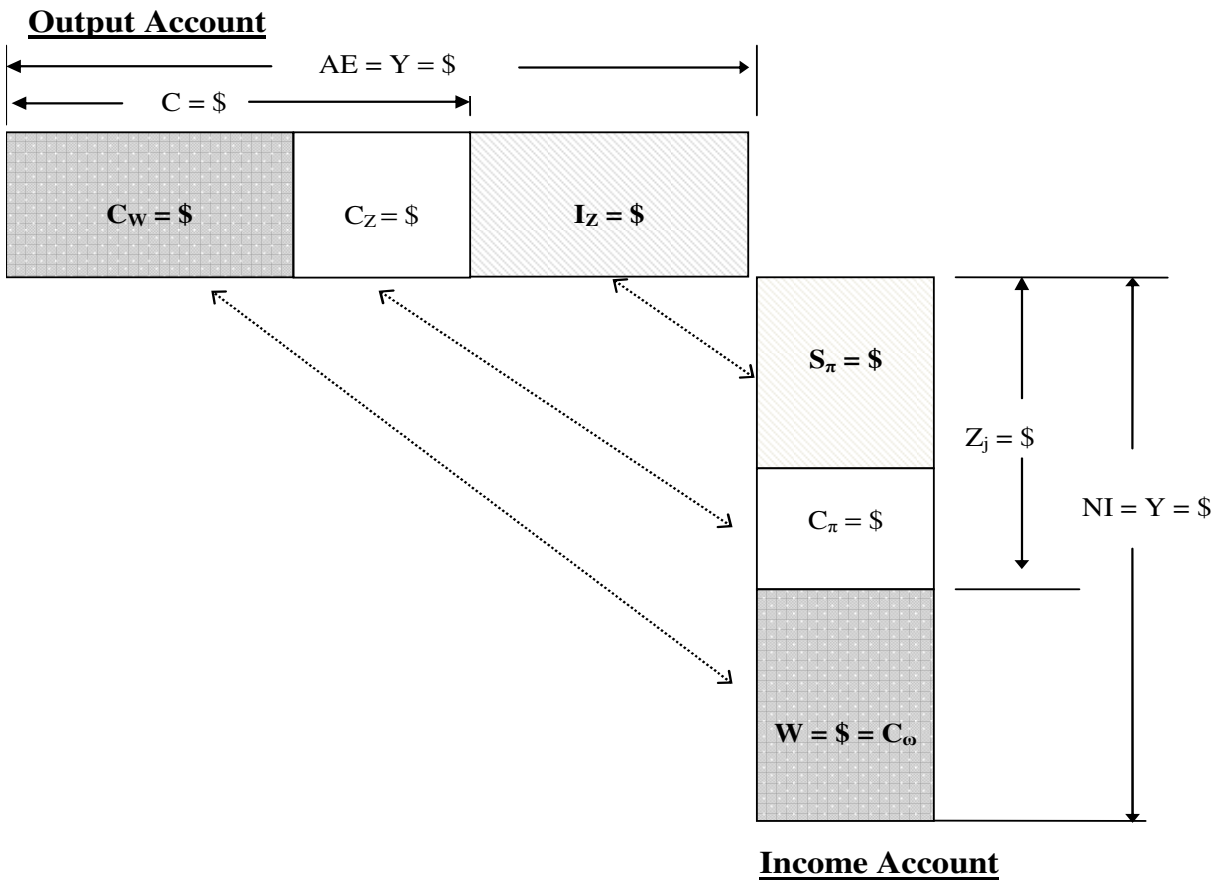
$$W = \omega Y = \text{aggregate wages} = \$$$

$$Z = (1 - \omega)Y = \pi Y = \text{aggregate profits} = \$$$

By definition the sum of wages and profits equal the aggregate net (surplus) product, and the identity showing this equality is referred to as the *income account* (Foley and Michl, 1999, Chapter 2) and is seen in the first column of Table 2 above. Distributive wage and profit revenues must be mapped to the *output account* (Foley and Michl, 1999, Chapter 2) that shows the expenditures associated with each revenue form. In this simple model developed here, the easiest way to conceive this mapping between aggregate accounts is to assume a constant propensity to save, denoted β. In general the propensity to save will be a pure number between zero and one; when $\beta = 0$ the propensity to save is zero and all revenues are consumed as expenditure. When $\beta = 1$ the propensity to save is unity and all revenues are saved for subsequent net investment. The propensity to consume is equal to one minus the propensity to save. Savings on the income account of the ledger will be mapped to investment expenditures on the output side of the ledger; alternatively consumption revenues on the

income-side will be mapped to consumption expenditures on the output side. This mapping of the surplus product between the different accounts can be expressed in terms of the following simple diagrammatic depiction[20]:

Figure 4: Diagrammatic Depiction of Mapping of Surplus Product

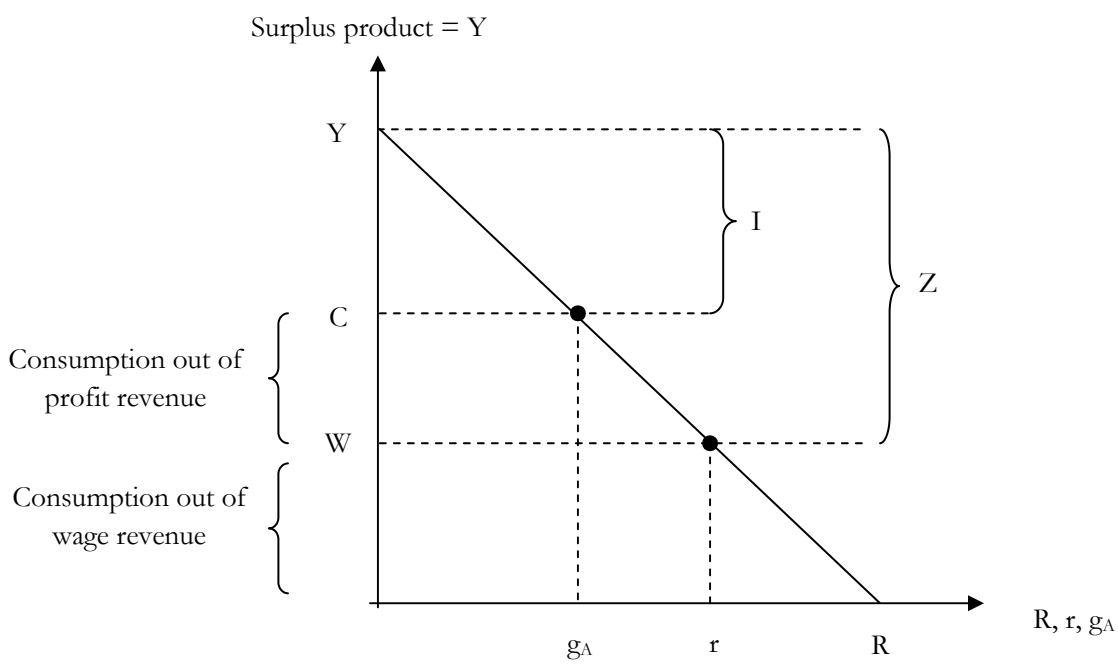


The relationship between the different expressions of the surplus product, viz. the income account expression as compared to the output account expression, can be formalized in the simple mathematical distribution-growth model as inspired by Foley and Michl (1999)[21] (recall we are assumming that depreciation is 100%):

$$\begin{aligned} &\text{Income Account:} \quad W = Y - (r)\,pA \\ &\text{Output Account:} \quad C = Y - (g_A)\,pA \end{aligned} \tag{7}$$

Note that g_A is the rate of growth of the means of production, or *accumulation rate of capital*, which we return to below. The income account and the output account so conceived can be drawn in output-profit rate space such that each account is expressed by the same schedule, referred to as the *growth-distribution schedule* (Foley and Michl, 1999: 29-31). The vertical axis of this schedule will show the net (surplus) product as distributed on both the income account side (W + Z) and the output account side (C + I) of the ledger. The horizontal axis will show the rates of profit (r) and growth (g_A) and will intersect at the maximum rate of profit (R) associated with a zero wage-share. This is shown in the following diagram:

Figure 5: Growth-distribution schedule



The growth rate of the means of production, or accumulation rate of capital (g_A), is in this simple framework equal to the profit portion of surplus product saved as revenues for investment expenditures (I) divided by the original value of the means of production (**pA**):

$$g_A = \frac{I}{pA} \tag{8}$$

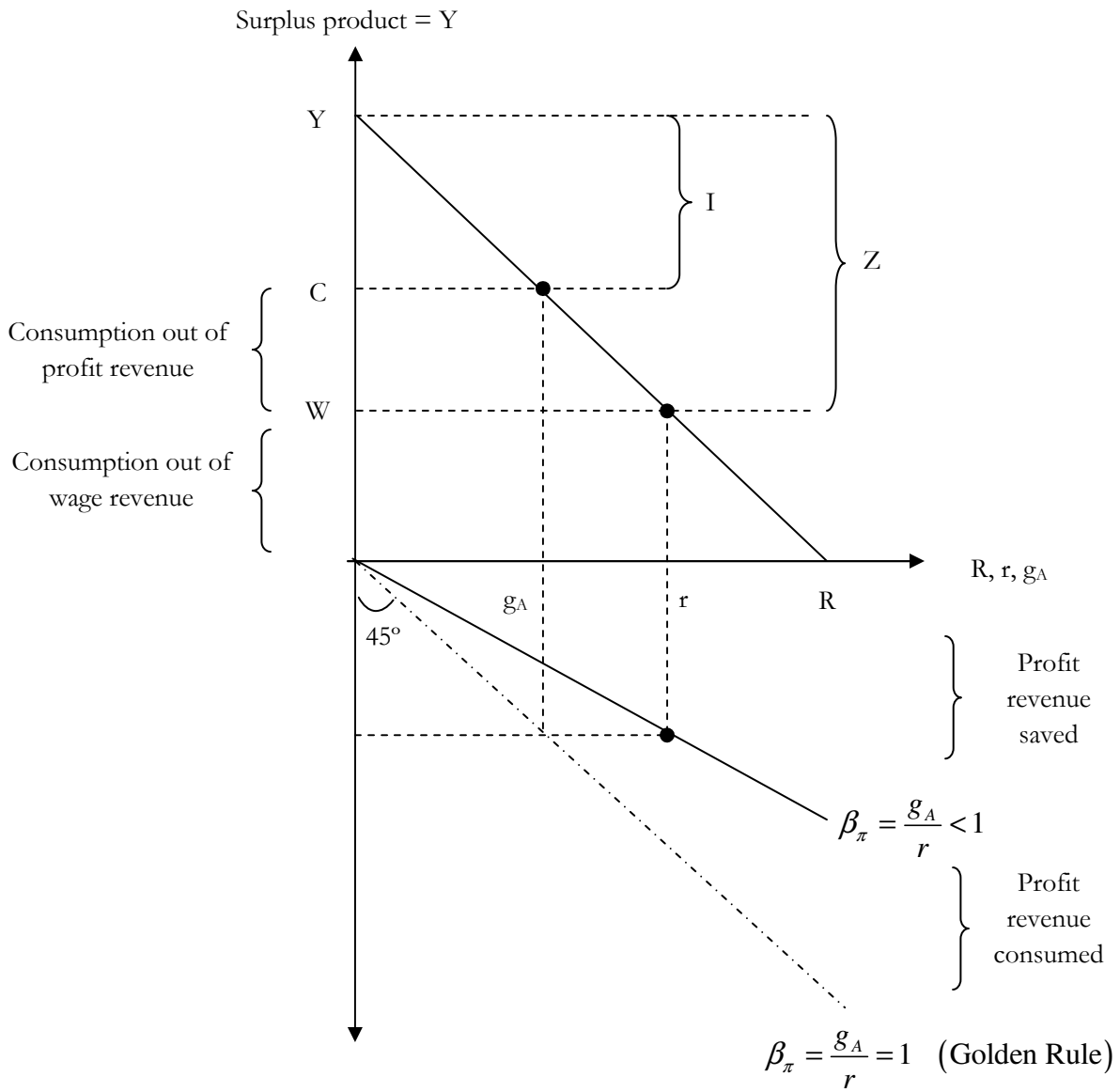
Recall from the mapping between the different aggregate accounts that the level of investment is equal to the propensity to save out of profits multiplied by the profit revenue (Z); hence the growth rate can be alternatively expressed as:

$$g_A = \frac{\beta_\pi Z}{pA} = \beta_\pi r_j \quad \Leftrightarrow \quad r_j = \frac{g_A}{\beta_\pi} \quad \Leftrightarrow \quad \beta_\pi = \frac{g_A}{r_j} \tag{9}$$

This equation states simply that the growth rate is equal to the savings propensity out of profit revenue multiplied by the rate of profit, and the profit rate is equal to the growth rate divided by this same propensity to save. This is nothing more than the simplest expression of the Cambridge Equation, an equation originating with Pasinetti (1962) that expresses the functional relation between profitability and growth.[22] When the propensity to save out of profit revenue is unity, all profits are saved for investment and the rate of growth is equal to rate of profit ; i.e. $(g_A)_{\beta=1} = r$. Alternatively when the savings propensity is zero, profit earners invest nothing and consume wholesale their revenue. This case corresponds to Marx's model of simple reproduction. When the wage share is zero, the profit rate is at its maximum value R, and if the savings propensity out of profits is equal to unity, then the growth rate too is at its maximum value corresponding to R. This maximum rate of growth has been called the "Golden Rule", and here profit earners are purely parsimonious and accordingly throw all of their revenue towards growth of the system.

Depicting the growth rate – profit rate Cambridge Equation nexus in Quadrant IV and combining it with the growth-distribution schedule, we can provide a more complete picture of a surplus-producing economy expressed in terms of the income and output relations (a similar graph appears in Kurz and Salvadori 1995, p. 48):

Figure 6: Growth-distribution schedule and the savings propensity out of profit revenue



Conclusion

The surplus approach to value and distribution represents an alternative manner to the orthodox neoclassical modeling of modern capitalistic economic systems. The framework is robust and allows for income account relations and output account relations to be put into relation with each other such that a holistic interpretation of the economy can be expounded and developed. The robustness of the framework allows for different conceptions of causality to be advanced; hence both Classical closure emanating from the income account side of the socio-economic ledger as well as Post Keynesian closure emanating from the output account of the ledger can equally be advanced and developed. The openness and robustness of the framework allows for the different theoretical traditions within heterodox economics to be put to the test vis-à-vis empirics, with the end result that the development of the laws of motion of modern capitalistic socio-economic systems can at last be understood and progressive policies aimed at addressing the exploitive and oppressive nature of these systems can be articulated and developed.

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[2] This essay deals exclusively with circulating capital models in which the means of production requirements are completely exhausted in the single round of production and the rate of depreciation is assumed to be 100 per cent.

[3] The review in question is by Melvin Reder (1961) writing for the *American Economic Review*, and the reference is Phillip Wicksteed (1914), whom Sraffa refers to as the "purist of marginal theory" (Sraffa, 1960, p. *v*).

[4] In his 1961 review of Sraffa's book for the *Scottish Journal of Political Economy*, Meek remarks that:

> "Mr. Sraffa's important book…can be looked at from various points of view. It can be regarded…simply as an unorthodox theoretical model of a particular type of economy, designed to solve the traditional problem of value in a new way. It can be regarded as an implicit attack on modern marginal analysis…[o]r finally it can be regarded as a sort of magnificent rehabilitation of the Classical (an up to a point Marxian) approach to certain crucial problems relating to value and distribution" (Meek, 1961, reprinted in Meek 1967: 161).

[5] I would like to thank Erik Olsen of the University of Missouri, Kansas City for making this distinction clear at the Workshop where the initial version of the present paper was presented.

[6] This is in marked distinction to the approach Sraffa adopted in Chapter VI of his book on the reduction to dated labor, where the analytics of the first five chapters that use the "physicalist" method are reproduced via the reduction methodology in this single chapter. In fact Sraffa heavily utilized the reduction framework in the 1940s prior to the arrival of the Standard system (see D3/12/21 Notes on Industrial Schemes of September 1942, D3/12/2 and D3/12/24 Notes 'Reduction' of October-December 1942; it was not until February 1944 that Sraffa finally discovers the properties of the Standard system and formally develops the *q*-system of quantity – read physicalist – multipliers; the

relevant folder is archived D3/12/36 and is simply entitled "Notes"). See Kurz and Salvadori (2008) for an account of Sraffa's forays of this time in the reduction methodology.

[7] This is actually a very significant point. Classical political economy, from Smith to Marx, has consistently utilized a comparative approach to the analysis of market-dominated economies through the juxtaposition of an "ideal" society of direct producer-owners who command the entirety of the net product resultant from their productivity. Smith referred to this as the "early and rude state" (Book I, Ch. iv); Ricardo and Malthus too discussed and debated this idea to the extent that both agreed that "labour commanded" coincided with "labour bestowed" in the "early and rude" society; Marx refers calls this simple or petty commodity production, and spends tremendous intellectual energy in the first three chapters of *Capital*, Volume I on the value form in this socio-economic system; and lastly Sraffa develops this through the physicalist analogue of the production for subsistence model in Chapter I of his book and expresses this socio-economic structure in value terms with the development in his unpublished notes of the "value theory of labour" (see below). What is beyond contention is that in this socio-economic system the labor theory of value holds. See Carter 2011 for further analysis of this important point.

[8] Pasinetti (1977, pp. 126-7) develops a formal model inclusive of the subsistence wage in terms of the "augmented A-matrix", i.e. the means of production matrix "'augmented' by the consumption coefficients needed for the maintenance of the workers".

[9] Matthew Forstater pointed out to me in 2009 that Sraffa's surplus production model is precisely analogous to the model in Quesnay's *Tableau* in the sense that it consists of "fertile" wheat production and "sterile" iron production.

[10] Sraffa continues:

> What is necessary are the given circumstances, i.e. the known ones (whether natural of social) of a given subject…Therefore, according to what an economist selects as the 'subject' of his economy (usually identifying himself with it), the 'surplus' will be different. The standpoint of capitalist society itself is that of the ruling class and therefore the surplus is composed of rent, interest and profit: Marx is the only economist who takes explicitly and consistently this point of view – and also Ricardo (specifically in Notes to Malthus) but not consistently. Marshall, who tries to take a classless human standpoint, regards all men as responsible subjects, and therefore all human consumption (he includes savings; this question of the inclusion of savings in income is also a question of who is 'subject') i.e. wages, interest, and rent as part of the surplus (which he calls the national dividend). Keynes, who takes the standpoint of the company director, regards only the 'entrepreneur'

(who is 'responsible for production') {as the subject, and} specifically defined 'profits' as surplus, all the factors having to be induced or paid according to contract (he goes as far as to regard past contracts as part of the given circumstances). Finally if one attempts to take an entirely objective point of view, the very conception of a surplus melts away. For if we take this natural science point of view, we must start by assuming that for every effect there must be a cause, that the causes are identical with the effects, and that there can be nothing in the effect which was not in the cause: in our case, there can be no product for which there has not been an equivalent cost, and all cost (= expenses) must be necessary to produce it" (D3/12/7/161 : 2-3).

[11] The importance of the Majorca Draft of 1955 has yet to be properly accounted for in the nascent literature on the Sraffa Papers (the same holds for Sraffa's archival activity in the 1950s generally speaking). The document itself, archived as D3/12/52, is a fascinating 31 page handwritten first draft of Part I of his book (single product industries).

[12] The reference to "insurmountable obstacle" is actually a reference to the term coined in a different context by Wicksell (1934, Volume II, p. 221). Sraffa employs this term repeatedly throughout his archival notes mostly in terms of the "value theory of labour" (see below).

[13] In the present essay we do not enter into the thorny issue of the determination of the money form of value and simply make the assumption that it can be determined, say by the simple assumption of commodity money the product of which is the result of "unassisted labour" that results from "silver picked up on the sea-shore in a day" (*Works XI* p. 94). The latter was a significant point of debate between Ricardo and Malthus (See *Works II*, p. 81; see also Carter 2011b). A major limitation of this assumption is the implicit neutrality of "money" in the present framework; hence "money" here serves simply as a measure of generalized exchange and the "universal" numeraire.

[14] The issue of the organic composition of the wage good sector has plagued Marxian economics since its inception, especially around the problem of Marx's transformation. This was especially brought to light by Wilhelm Lexis in a review of *Capital*, Volume III, that was published in the *Quarterly Journal of Economics* in 1895 (Lexis 1895) Sraffa mentions Lexis on several occasions in his papers, speaking when addressing the wage good sector of the "'Lexis' consideration" in the Majorca Draft (D3/12/52/5). See Howard and King, 1989, p. 47 for a discussion on Lexis regarding this very point. Garegnani (1984, Section VI, pp. 313-320)) develops some of the analytics of this with the construction of the "vertically-integrated wage-good sector", conceived as an abstract subsystem within a larger economic system of both basics and non-basics around which the rate of profit for the entire system is determined. In essence he constructs a vertically-integrated "corn model", where Ricardo's "corn" is replaced by Garegnani's wage good

sector (he admits as much on page 313, note 39). He then juxtaposes this treatment to Sraffa's own construct of the Standard commodity and highlights the resonance between both approaches (see Section VII).

[15] The Marxist critique is found in de Brunhoff (1974-5) and Laibman (1974-5), and Sraffian in Roncaglia (1978); Kurz and Salvadori (1987); Garegnani (1984); and Pasinittei (1977). Sinha (2000) also discusses Eatwell's approach with disapproval.

[16] The analysis that follows is strictly conducted in terms of the aggregates of given quantities that Sraffa himself utilized in his book. Hence the equations express the physical levels of output, labor, and money-as-numeraire magnitudes, i.e. they are *not* unit input-output coefficients that are divided by gross output, etc. Hence no questions of returns to scale need apply (see Kurz and Salvadori 1995, p. 43).

[17] Pasinetti's notation does reflect the more conventional inter-industry framework, with the matrix **A** equal to the inter-industry coefficients matrix, the (row) vector $\mathbf{a_n}$ equal to the direct labor requirements, the (row) vector **p** equal to the (no-distribution) set of prices, and the scalar value *w* the wage rate.

[18] The extreme case of an economic system containing only one fertile industry is that of Ricardo's corn model, especially as Sraffa interprets the 1815 *Essay on Profits*. (see Sraffa's Introduction to *Works I*, Sarffa and Dobb, 1951, pp. *xxx-xxxvii*).

[19] On the resonance between the New Interpretation and Sraffa see an important paper by Stefano Perri (2010).

[20] Although in somewhat modified form, the present diagrammatic depiction between the income and output account is inspired by similar (though not exact) diagrams, of increasing complexity, in Shaikh and Tonak (1994).

[21] The present distribution-growth model differs from Foley and Michl (1999) in that ours does not divide each account by the number of workers; i.e. ours are "$" magnitudes whereas Foley and Michl's are "$/_{wkr}$" magnitudes.

[22] Foley and Michl (1999; Chapter 5) develop a more sophisticated model of consumption, savings and investment through the utilization of an inter-temporal logarithmic discounted Cobb-Douglas utility function subject to the inter-temporal budget constraint of current consumption and next-period capital accumulation.